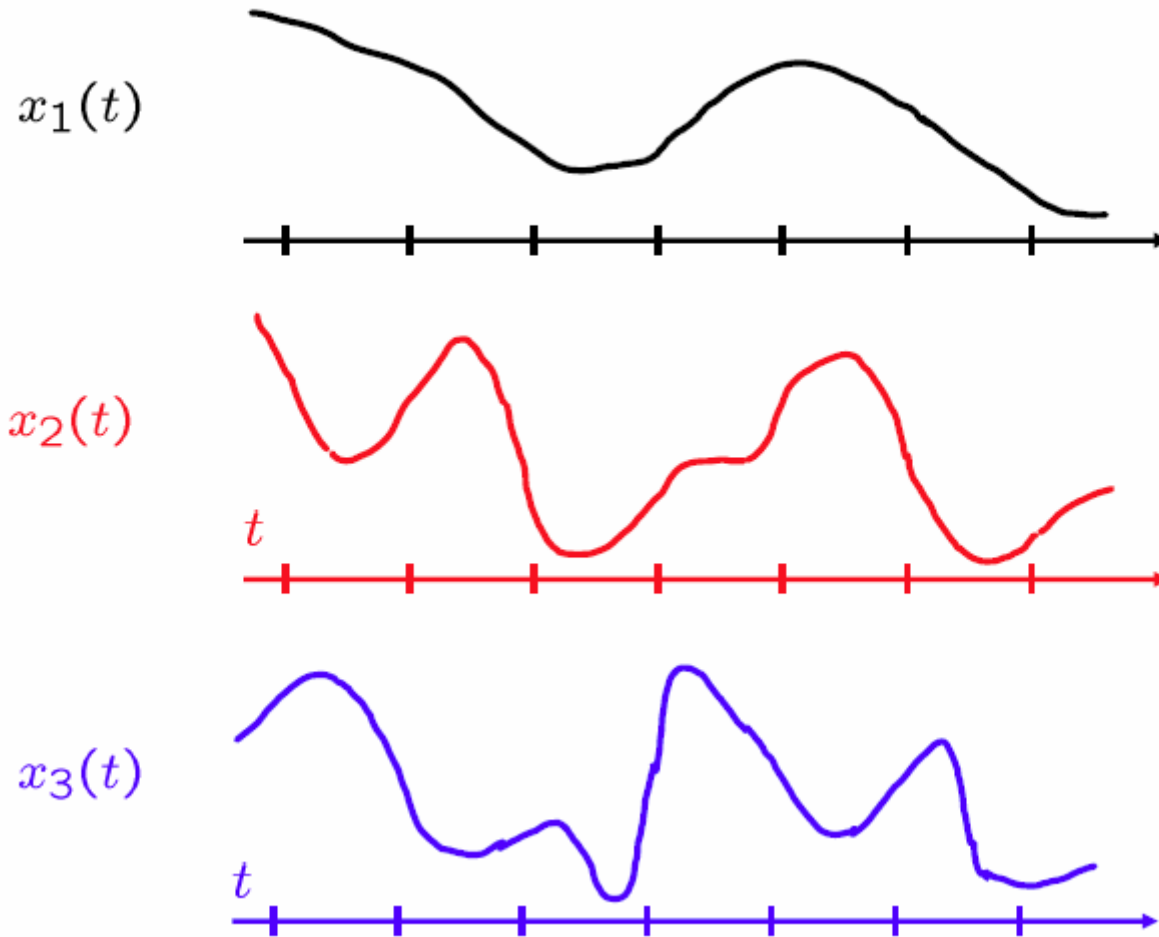


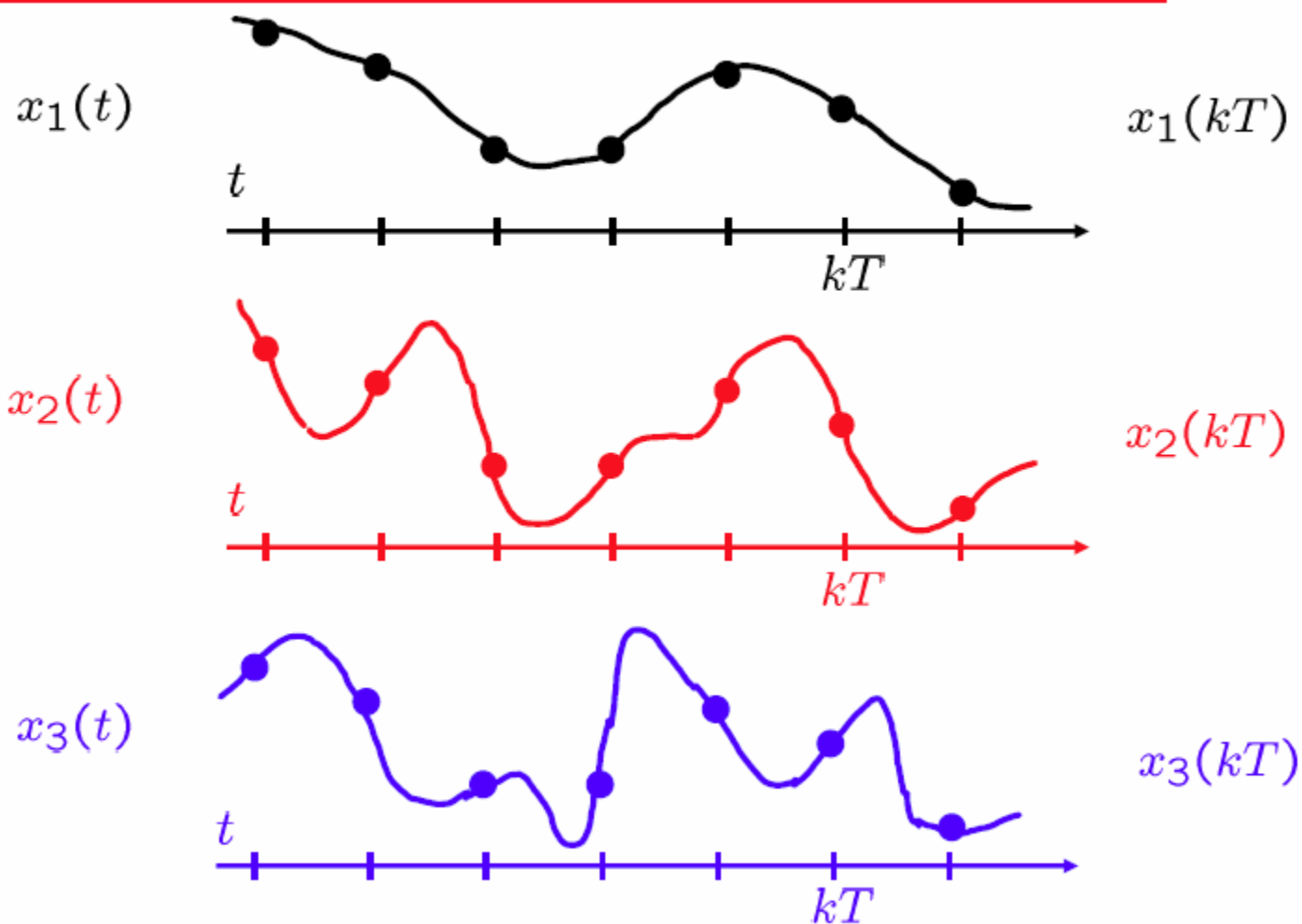
SAMPLING

- Representation of of a Continuous-Time Signal by Its Samples: The Sampling Theorem
- Reconstruction of of a Signal from Its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

- Representation of CT Signals by its Samples

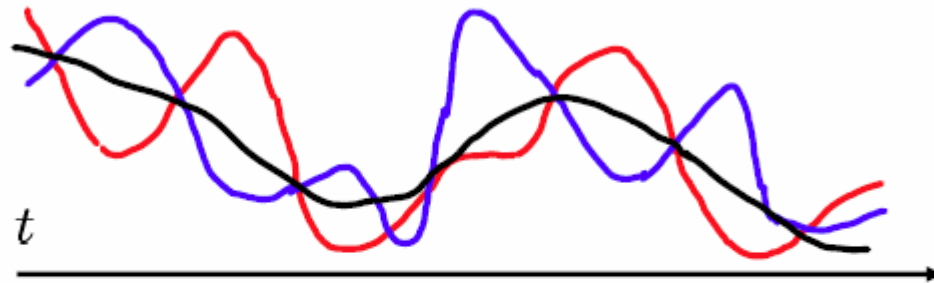


■ Representation of CT Signals by its Samples

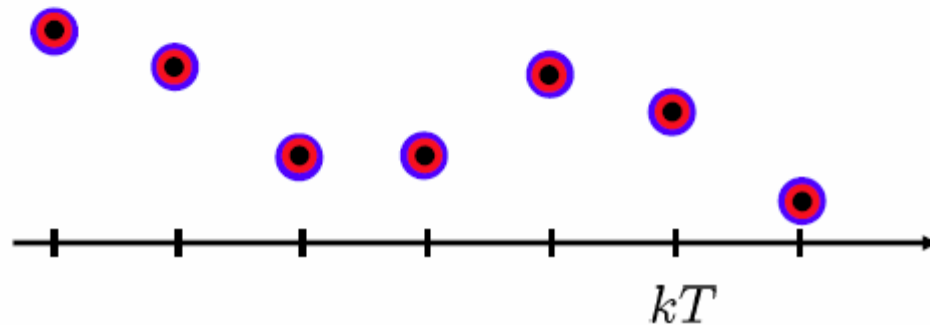


■ Representation of CT Signals by its Samples

$$x_1(t) \neq x_2(t) \neq x_3(t)$$



$$x_1(kT) = x_2(kT) = x_3(kT)$$



■ Impulse-Train Sampling:

$p(t)$: sampling function

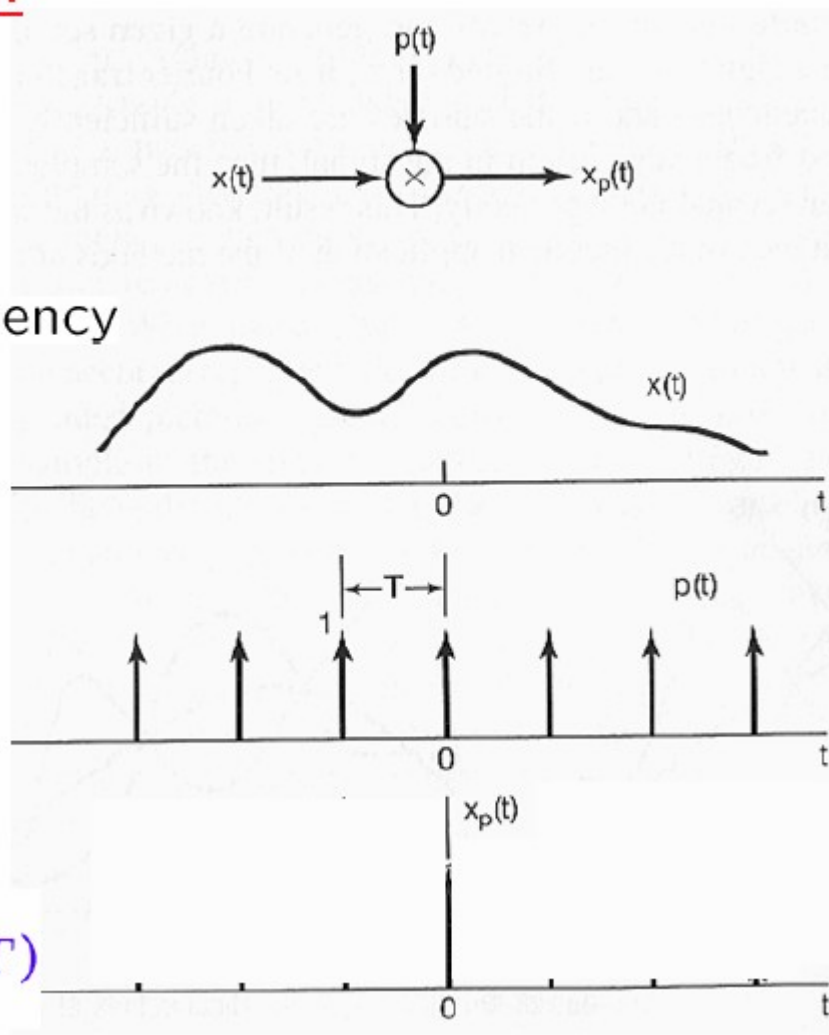
T : sampling period

$w_s = \frac{2\pi}{T}$: sampling frequency

$$\Rightarrow x_p(t) = x(t) p(t)$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t - nT)$$



- Impulse-Train Sampling: From multiplication property,

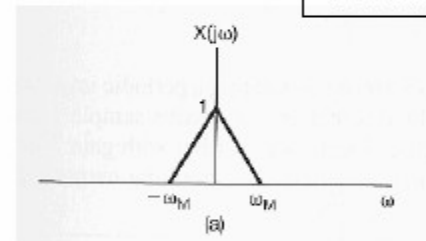
$$x_p(t) = x(t) p(t) \xleftrightarrow{\mathcal{F}} X_p(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta) P(j(\omega - \theta)) d\theta$$

Eq 4.70, p. 322

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s))$$

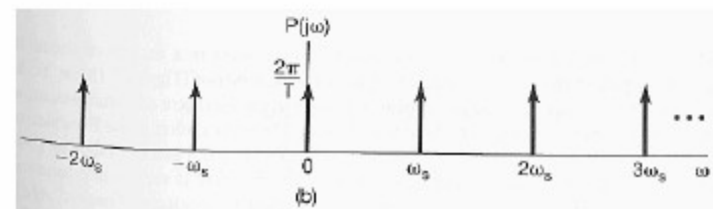
Ex 4.21, p. 323

$$x(t) \xleftrightarrow{\mathcal{F}} X(j\omega)$$



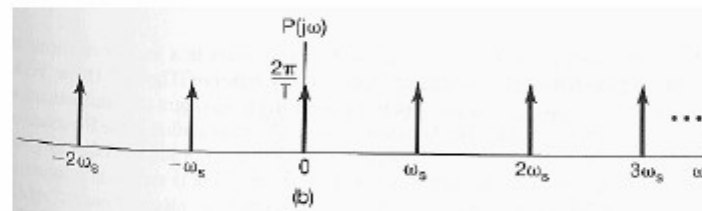
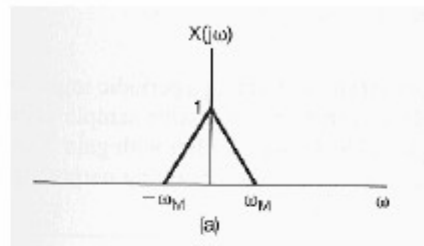
$$p(t) \xleftrightarrow{\mathcal{F}} P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_s)$$

Ex 4.8, pp. 299-300

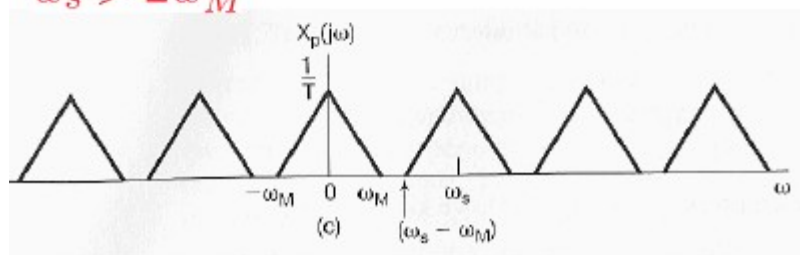


■ Impulse-Train Sampling:

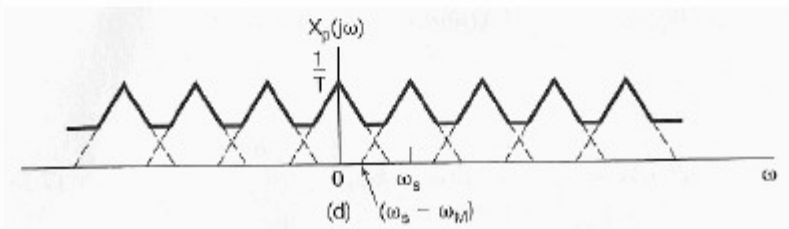
Ex 4.21, 4.22, pp. 323-4



$\omega_s > 2\omega_M$



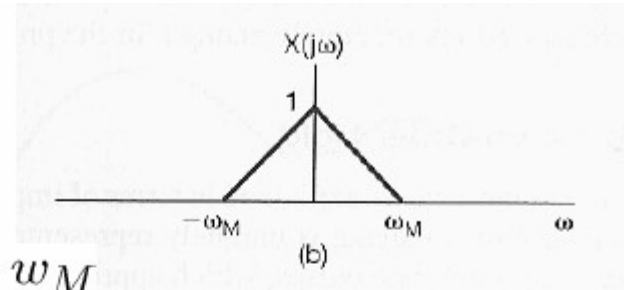
$\omega_s < 2\omega_M$



- The Sampling Theorem:

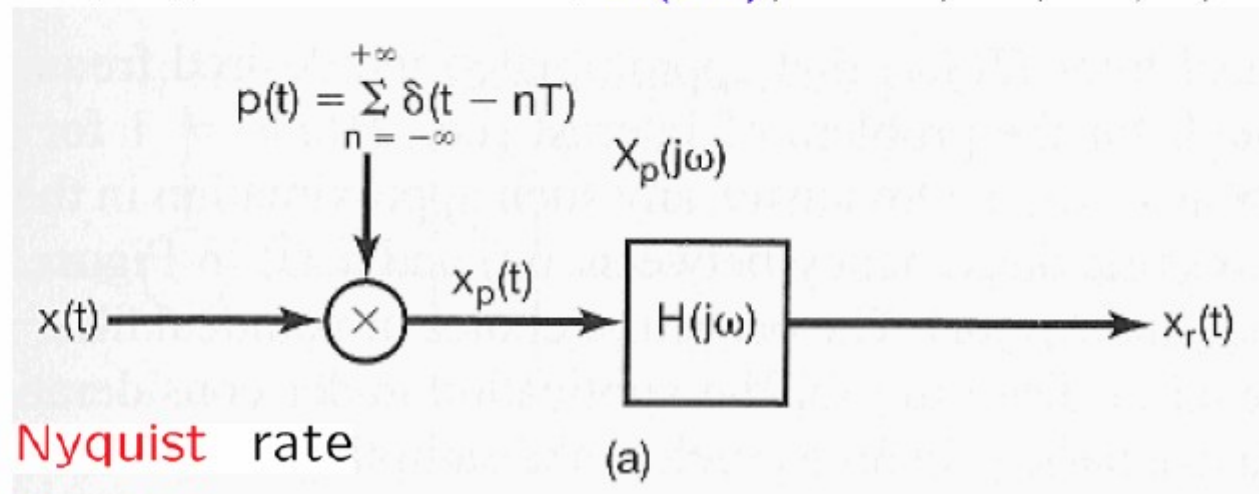
$x(t)$: a band-limited signal

with $X(j\omega) = 0$ for $|\omega| > \omega_M$



if $\omega_s > 2\omega_M$ where $\omega_s = \frac{2\pi}{T}$

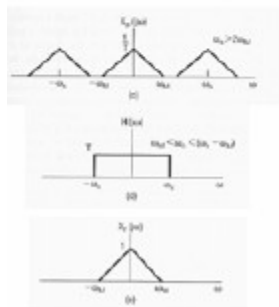
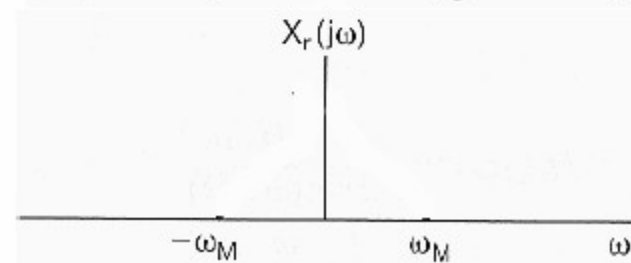
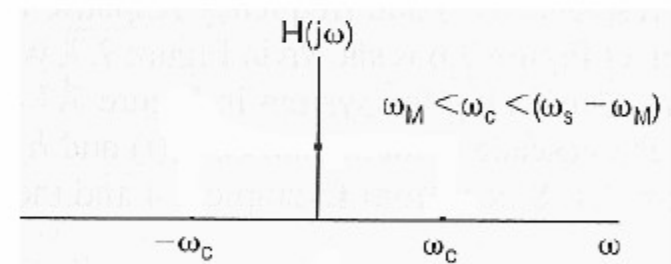
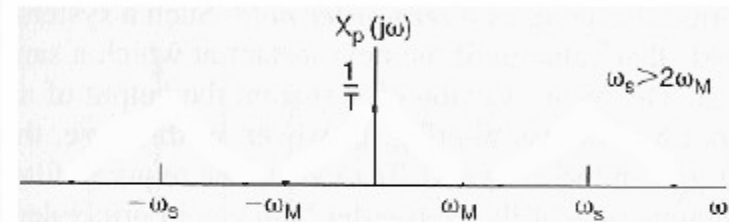
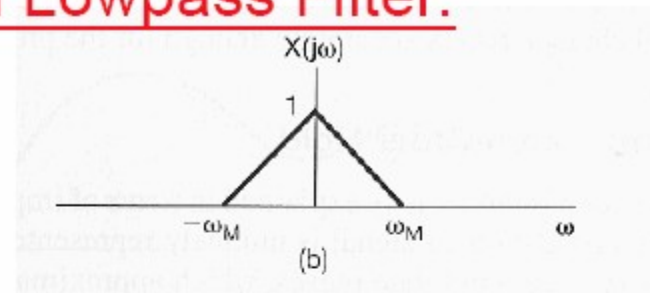
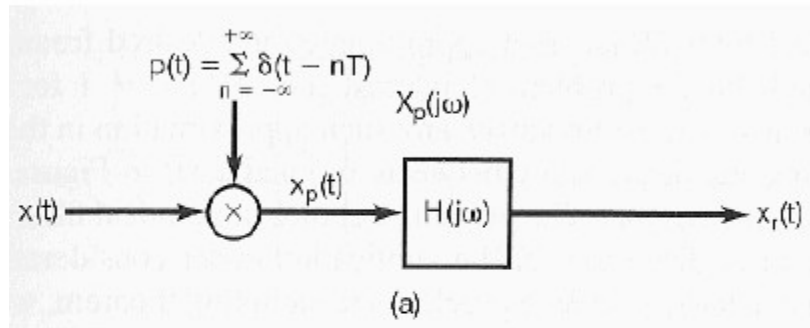
$\Rightarrow x(t)$ is uniquely determined by $x(nT)$, $n = 0, \pm 1, \pm 2, \dots$,



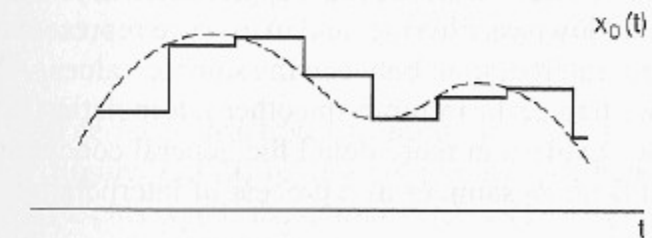
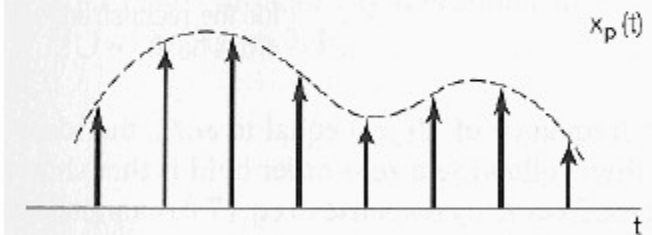
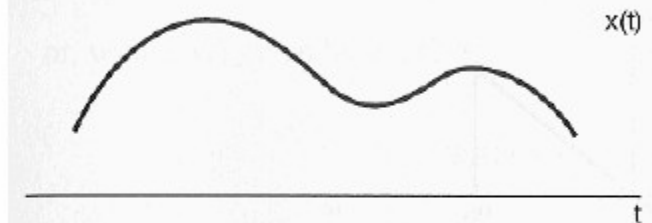
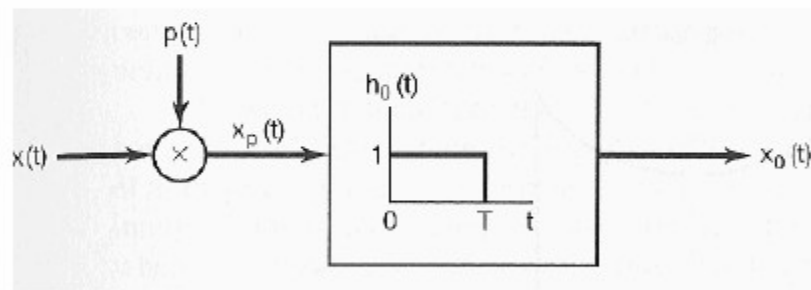
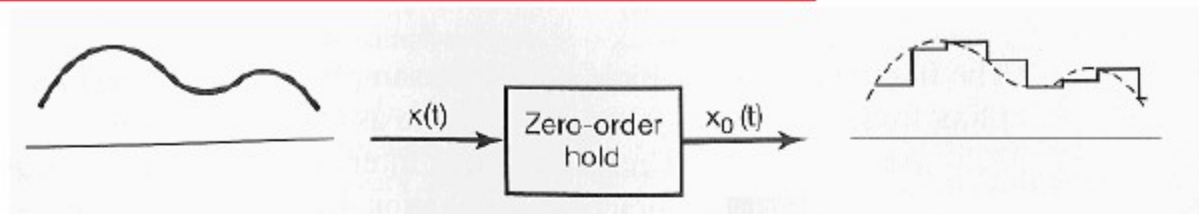
$\Rightarrow 2\omega_M$: **Nyquist** rate

ω_M : **Nyquist** frequency

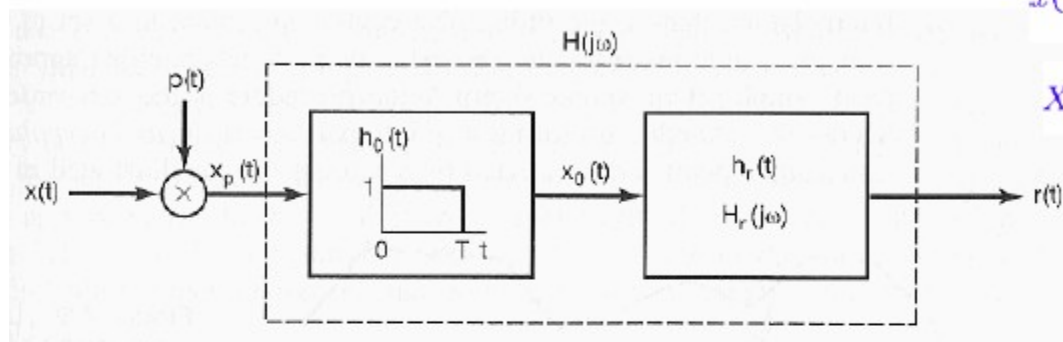
▪ Exact Recovery by an Ideal Lowpass Filter:



■ Sampling with Zero-Order Hold:



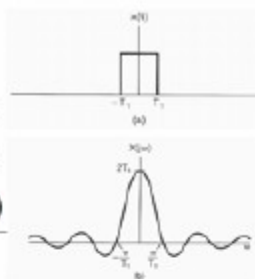
■ Sampling with Zero-Order Hold:



Ex 4.4, p. 293

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$

$$X(j\omega) = 2 \frac{\sin(\omega T_1)}{\omega}$$



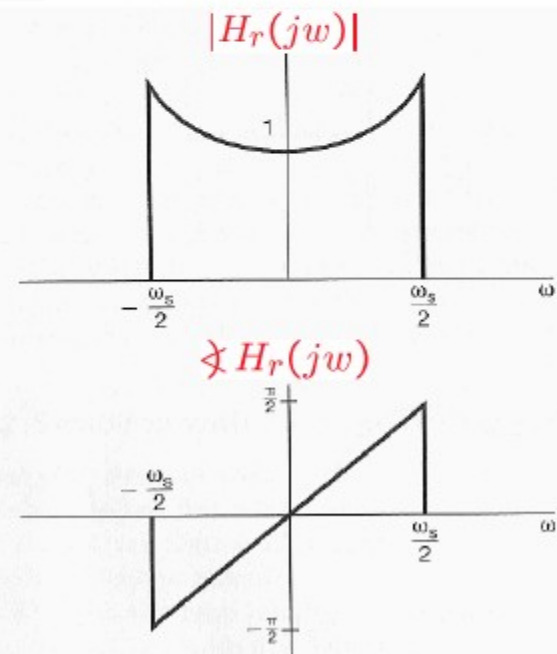
Eq 4.27, p. 301

$$x(t - t_0) \xleftrightarrow{\mathcal{F}} e^{-j\omega t_0} X(j\omega)$$

$$H_0(j\omega) = e^{-j\omega T/2} \left[\frac{2 \sin(\omega T/2)}{\omega} \right]$$

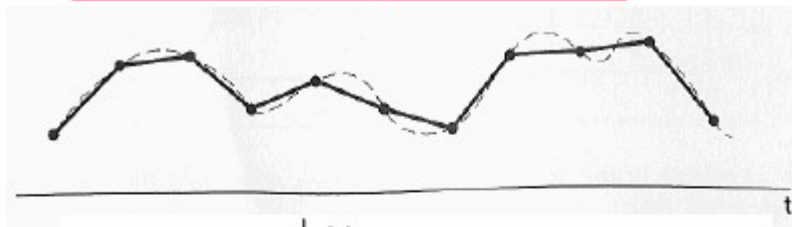
$$H(j\omega) = H_0(j\omega) H_r(j\omega)$$

$$\Rightarrow H_r(j\omega) = \frac{e^{j\omega T/2} H(j\omega)}{2 \sin(\omega T/2)}$$



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Exact Interpolation:



$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$

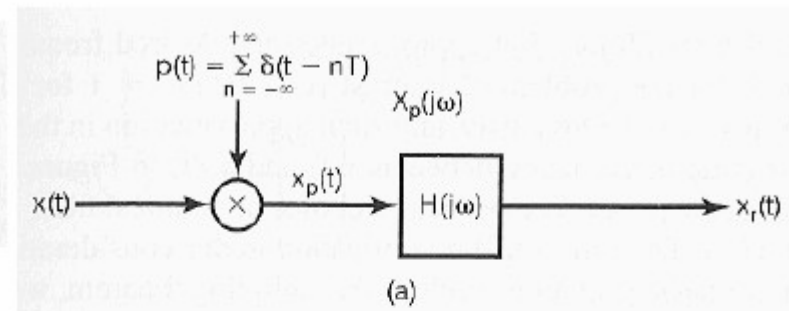
$$x_r(t) = x_p(t) * h(t)$$

Ex 2.11, p. 110

$$x(t - t_0) = x(t) * \delta(t - t_0)$$

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT)h(t - nT)$$

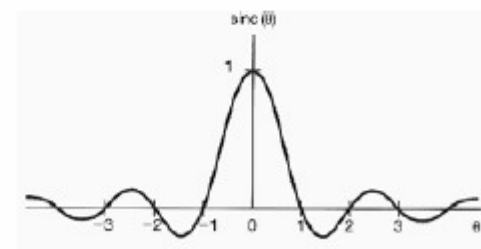
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \frac{w_c T}{\pi} \frac{\sin(w_c(t - nT))}{w_c(t - nT)}$$



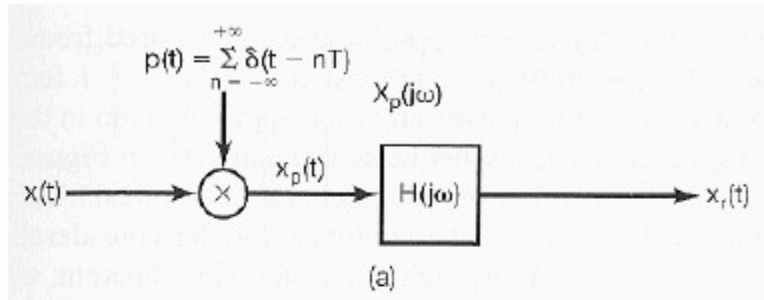
ideal lowpass filter

with a magnitude of T

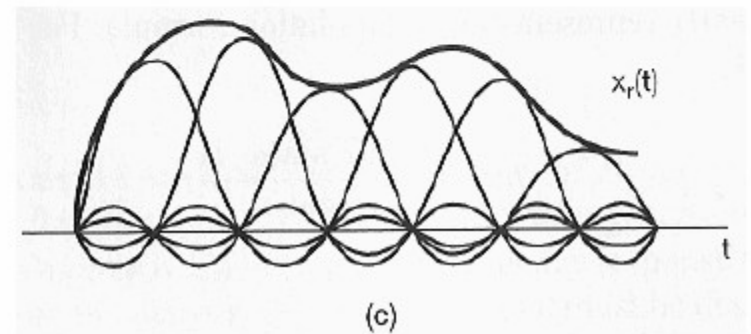
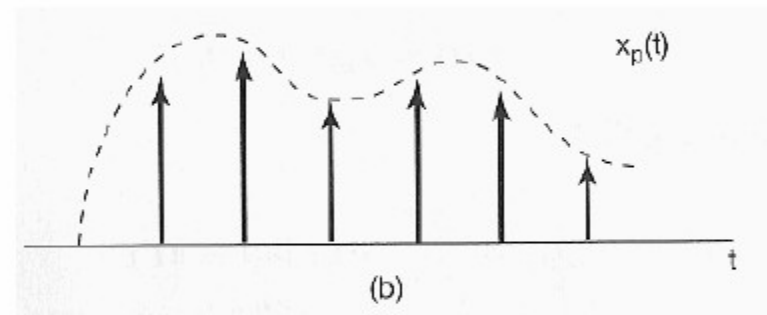
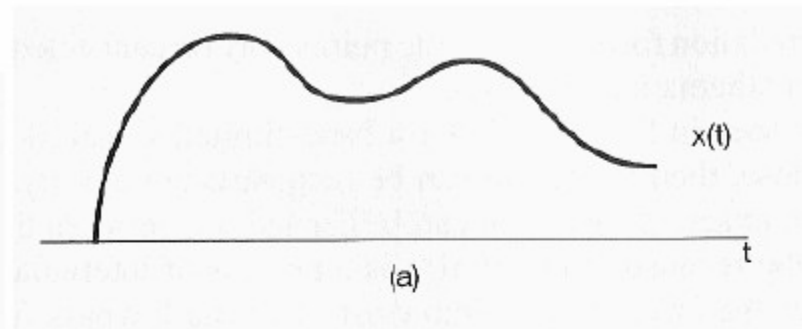
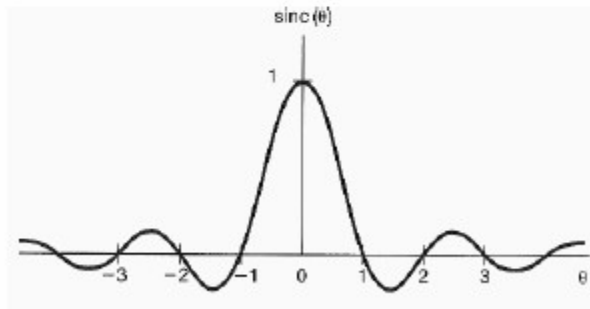
$$h(t) = T \frac{w_c}{\pi} \frac{\sin(w_c t)}{w_c t}$$



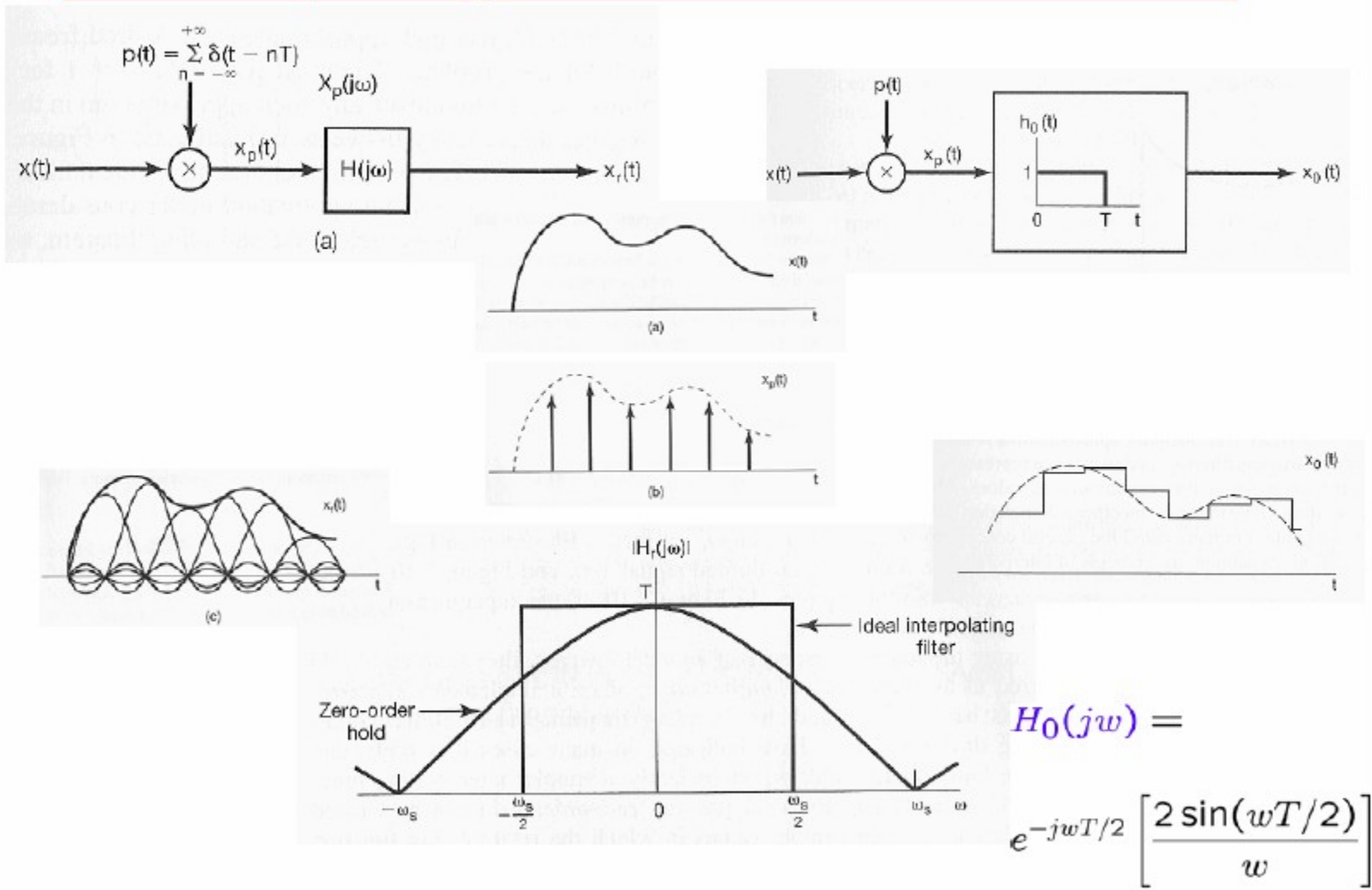
Exact Interpolation:



$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \frac{w_c T}{\pi} \frac{\sin(w_c(t - nT))}{w_c(t - nT)}$$



■ Ideal Interpolating Filter & The Zero-Order Hold:



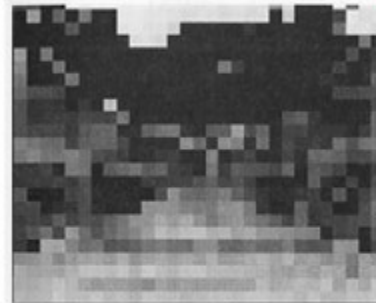
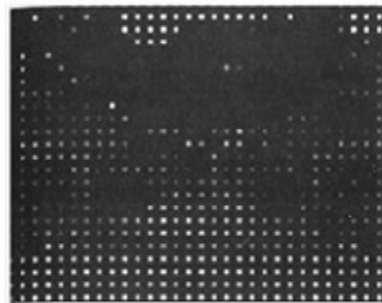
■ Sampling & Interpolation of Images:

original image impulse sampling zero-order hold zero-order hold

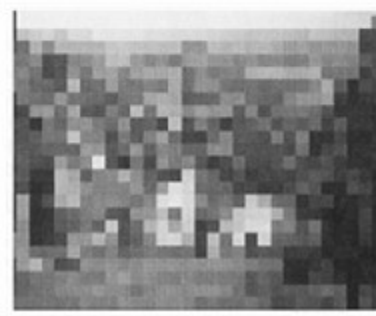
4 : 1



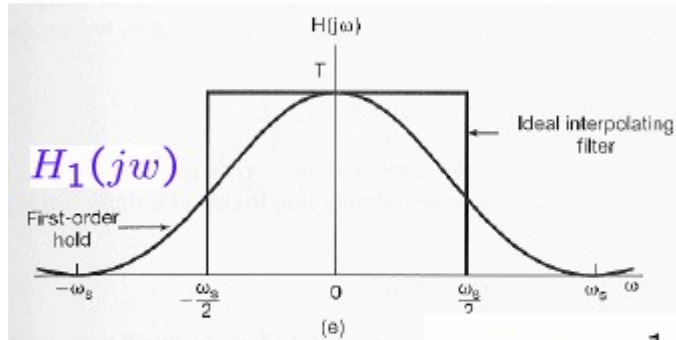
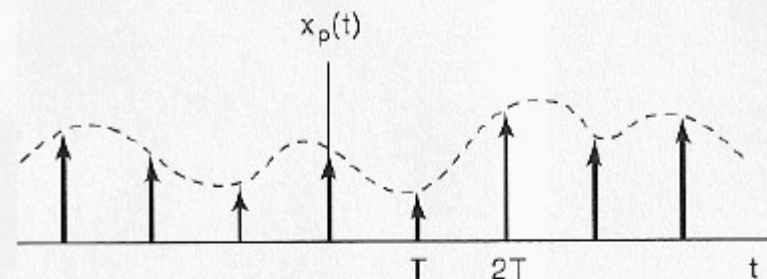
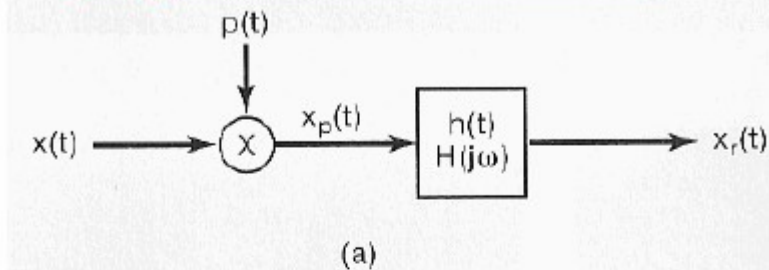
(a)



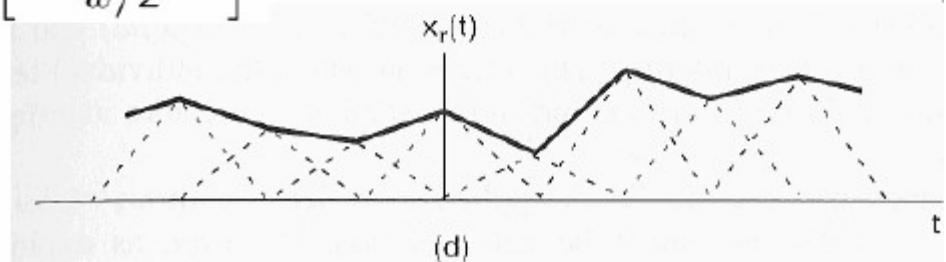
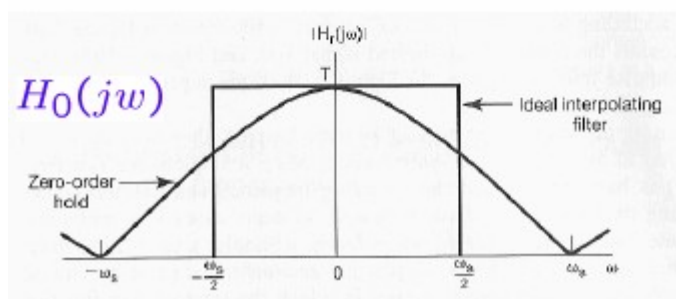
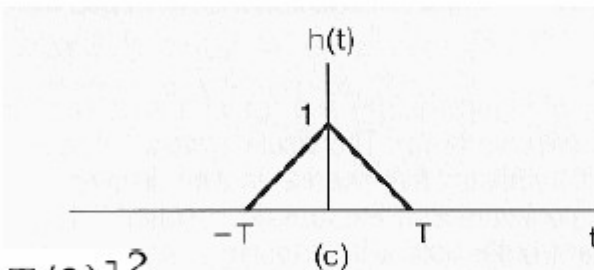
(c)



Higher-Order Holds:



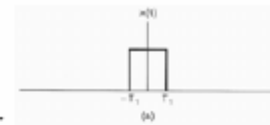
$$H_1(j\omega) = \frac{1}{T} \left[\frac{\sin(\omega T/2)}{\omega/2} \right]^2$$



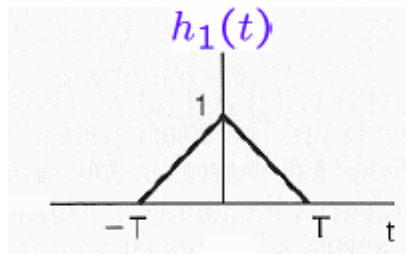
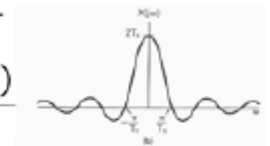
■ Higher-Order Holds:

Ex 4.4, p. 293

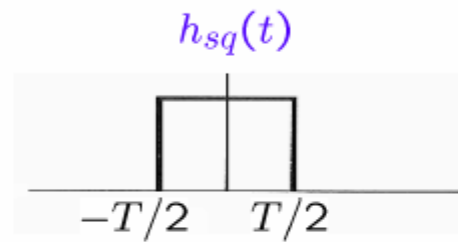
$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$



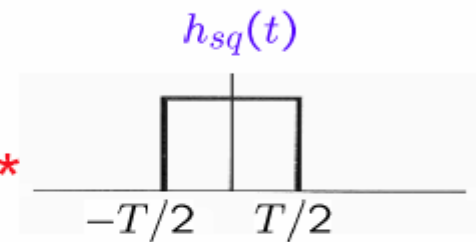
$$X(j\omega) = 2 \frac{\sin(\omega T_1)}{\omega}$$



$$= \frac{1}{T}$$



*



$$H_1(j\omega) = \frac{1}{T} \cdot 2 \frac{\sin(\omega T/2)}{\omega} \times 2 \frac{\sin(\omega T/2)}{\omega}$$

$$= \frac{1}{T} \left[\frac{\sin(\omega T/2)}{\omega/2} \right]^2$$

- First-Order Hold on Image Processing:

zero-order hold

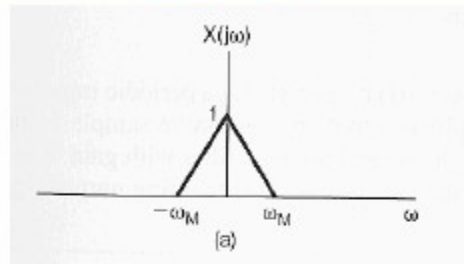


first-order hold

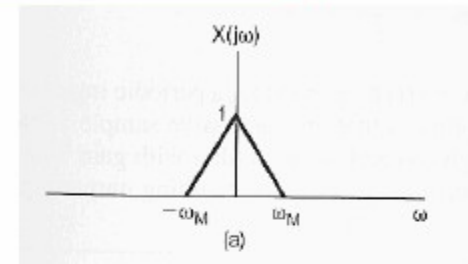
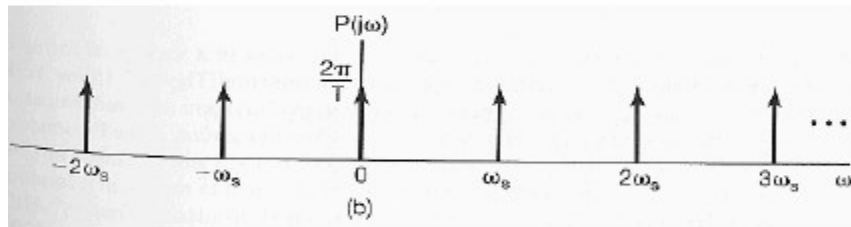


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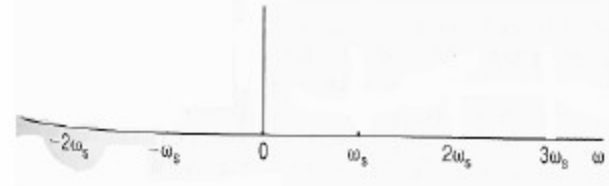
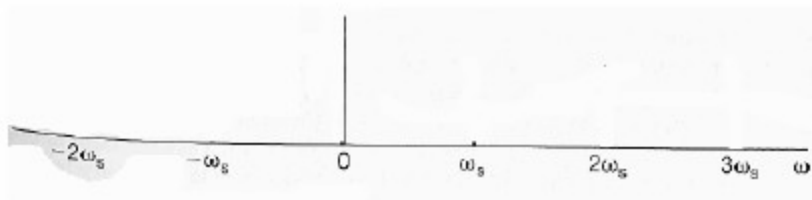
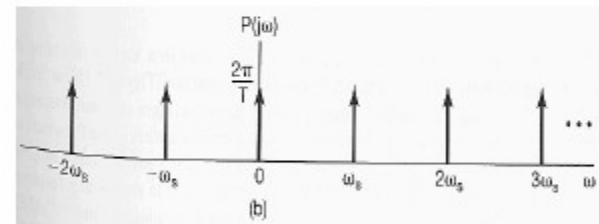
■ Overlapping in Frequency-Domain: Aliasing



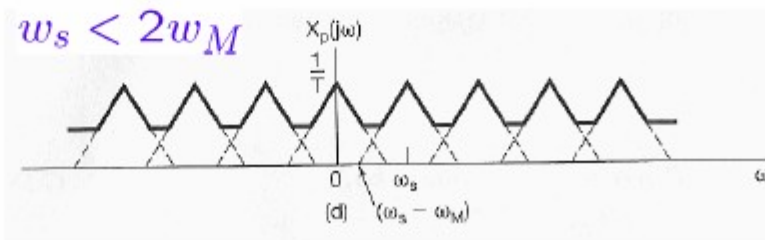
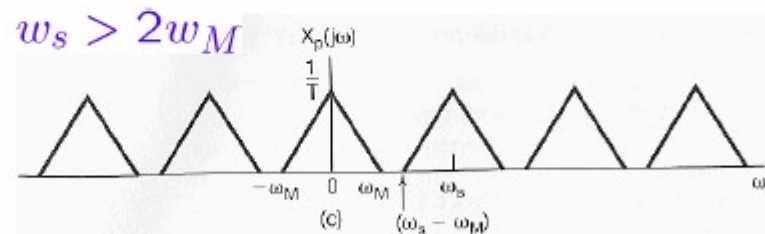
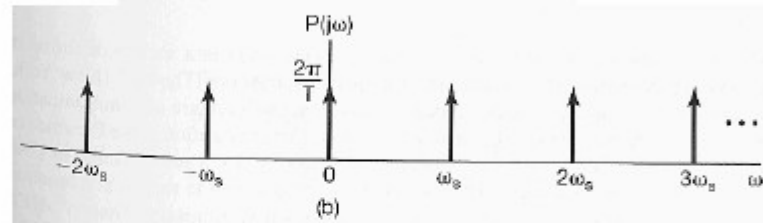
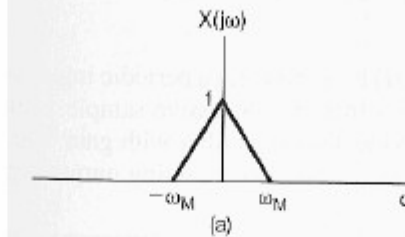
$$\omega_s > 2\omega_M$$



$$\omega_s < 2\omega_M$$



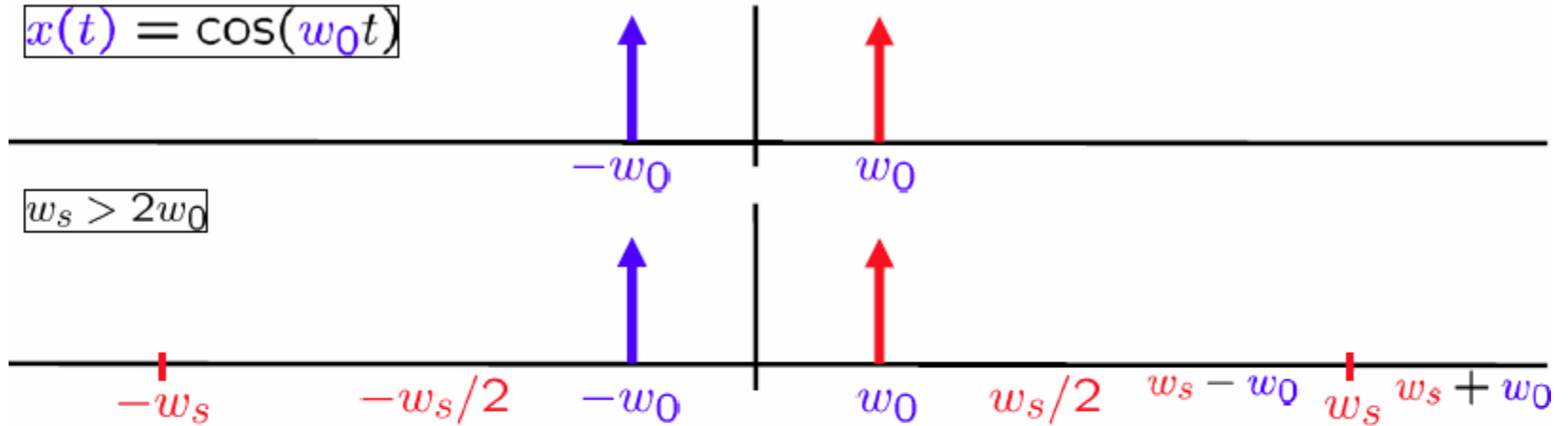
■ Overlapping in Frequency-Domain: Aliasing



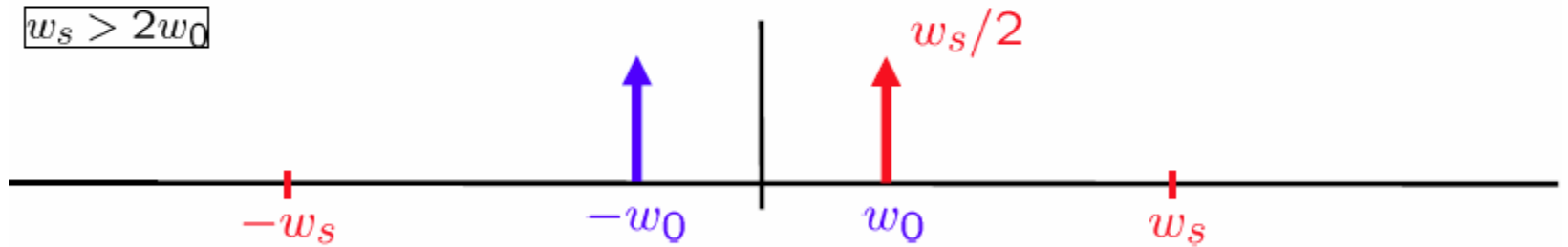
Overlapping in Frequency-Domain: Aliasing

$$x(t) = \cos(w_0 t)$$

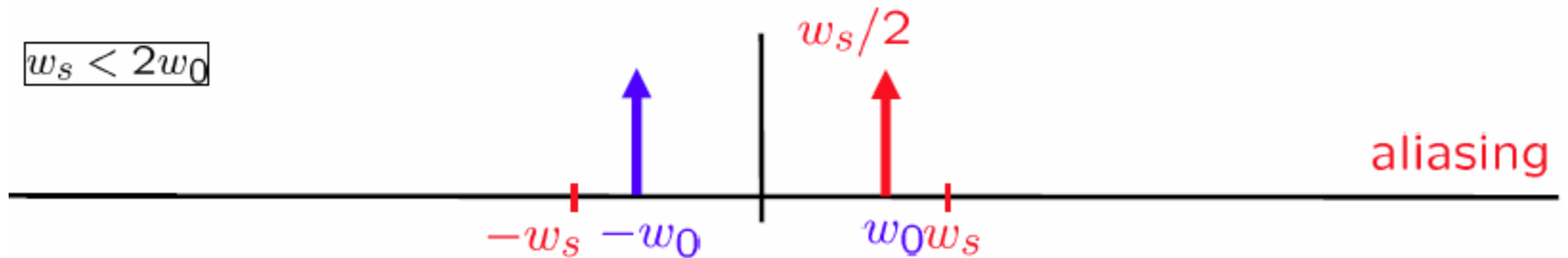
$$w_s > 2w_0$$



$$w_s > 2w_0$$

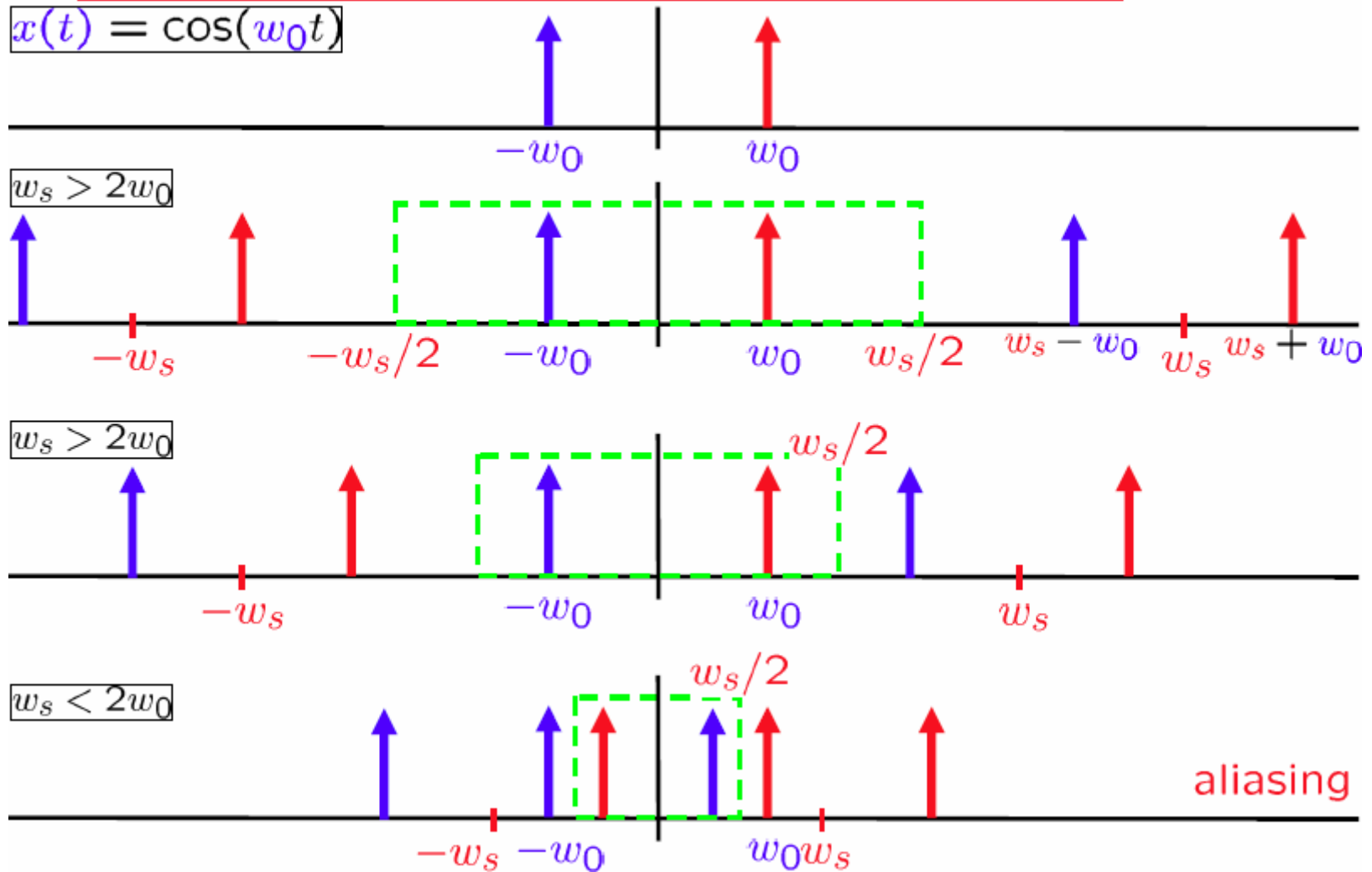


$$w_s < 2w_0$$



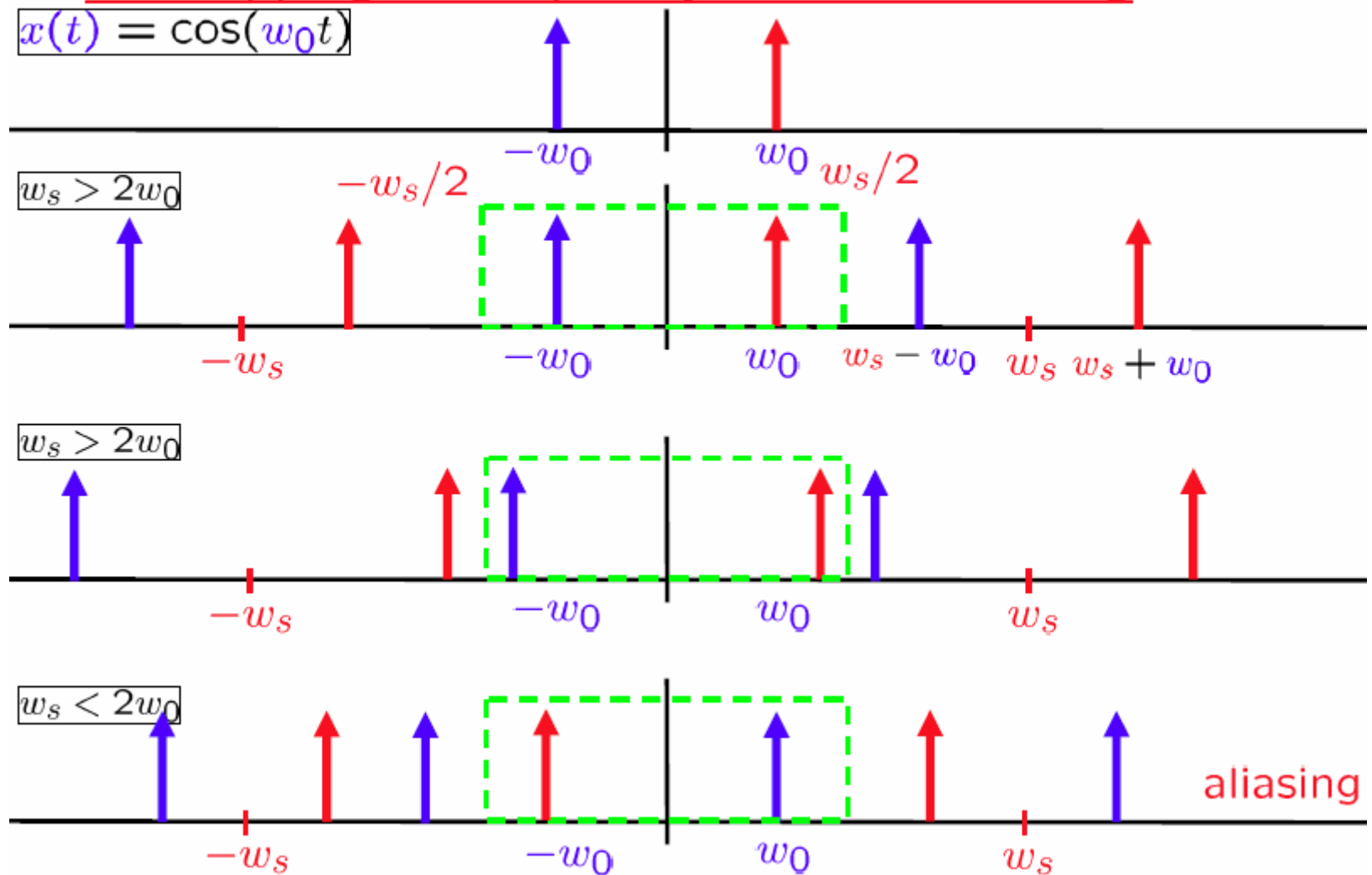
Overlapping in Frequency-Domain: Aliasing

$$x(t) = \cos(w_0 t)$$

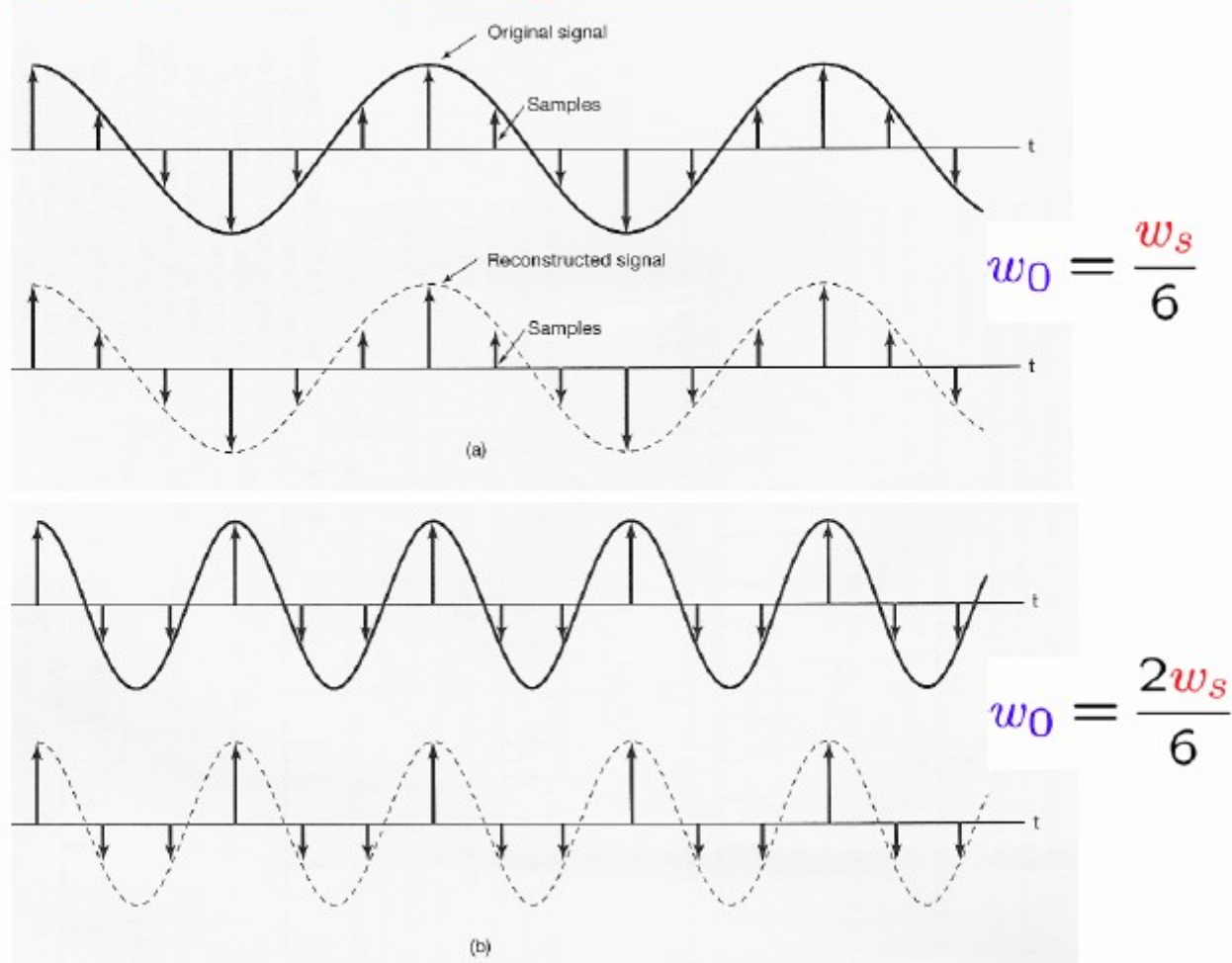


Overlapping in Frequency-Domain: Aliasing

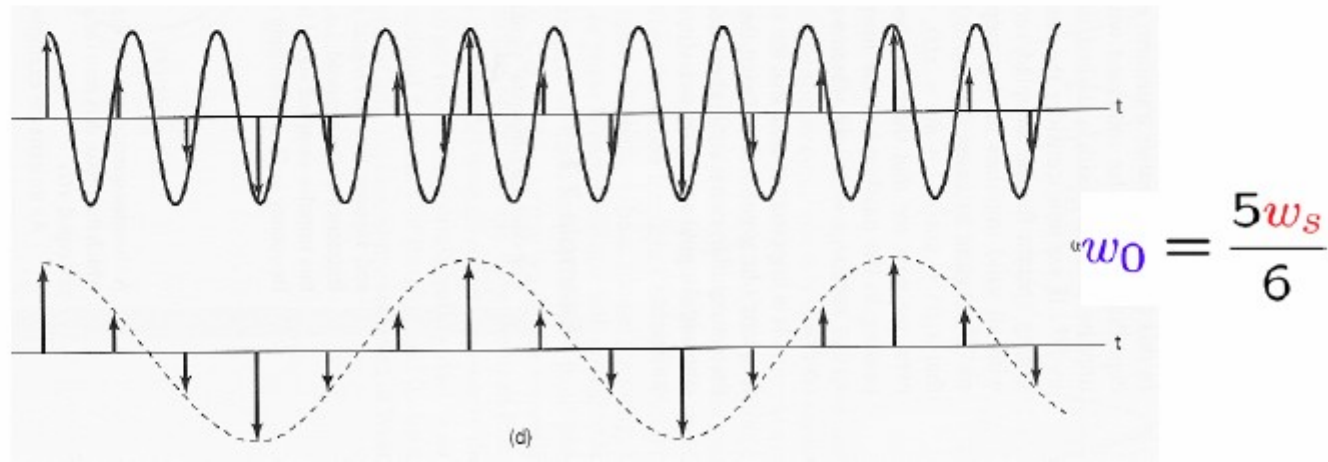
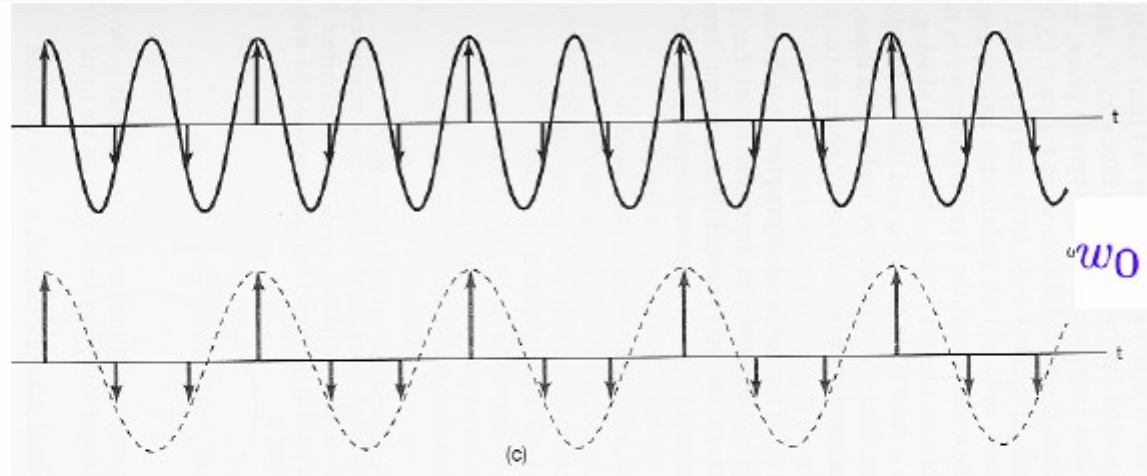
$$x(t) = \cos(w_0 t)$$



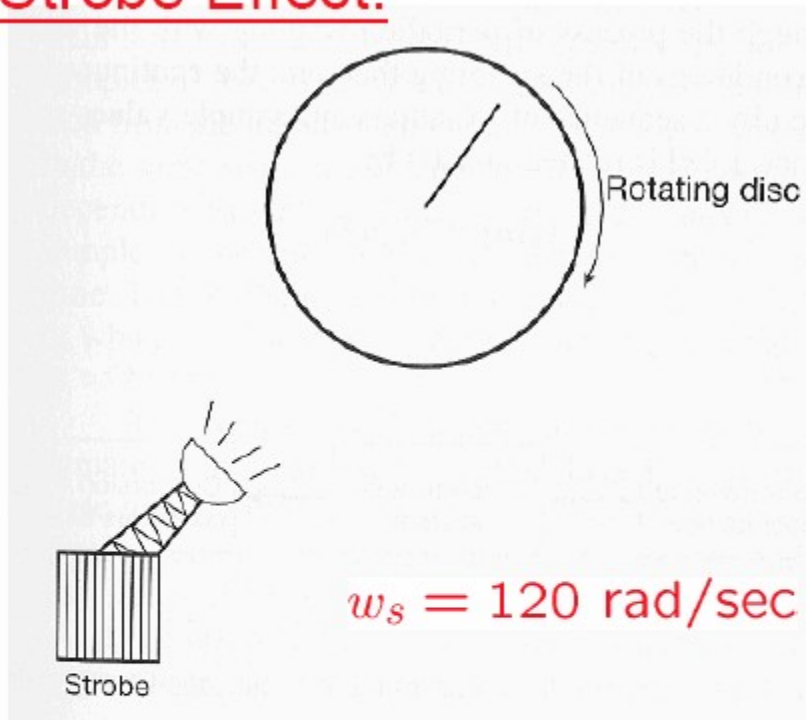
■ Overlapping in Frequency-Domain: Aliasing



■ Overlapping in Frequency-Domain: Aliasing



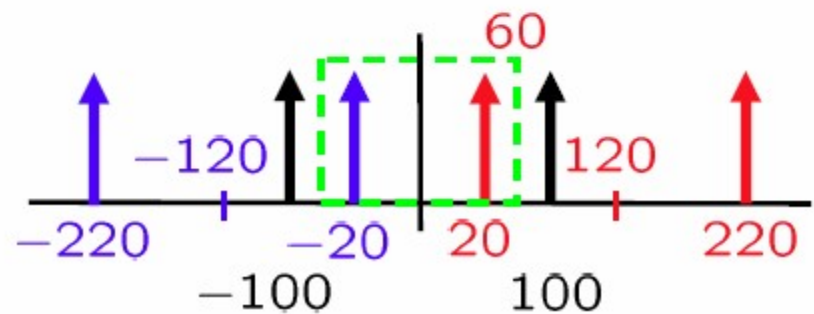
■ Strobe Effect:



$$w_0 = 100 \text{ rad/sec}$$

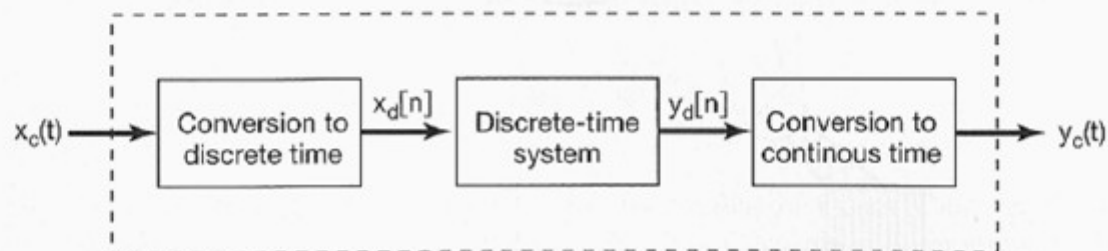
$$\Rightarrow w = \pm w_s \pm w_0$$

$$= +20, -20$$

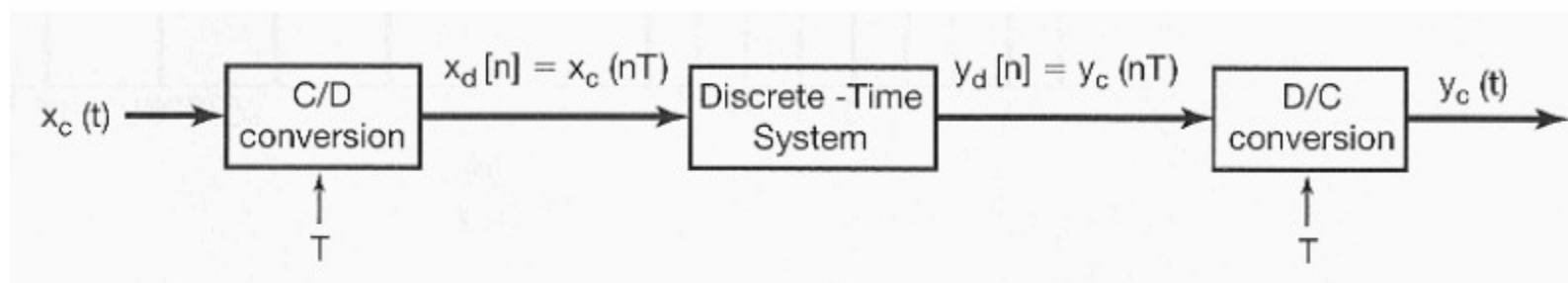


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- Discrete-Time Processing of CT Signals:



- C/D or A-to-D (ADC) and D/C or D-to-A (DAC):



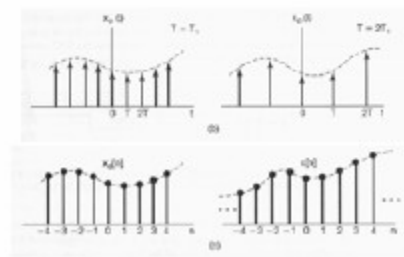
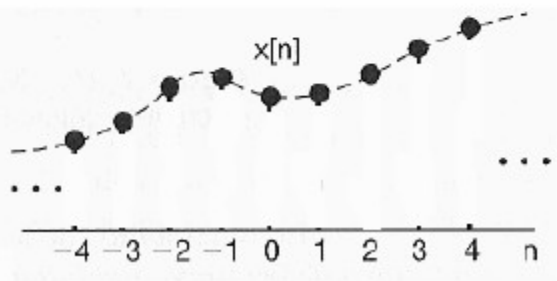
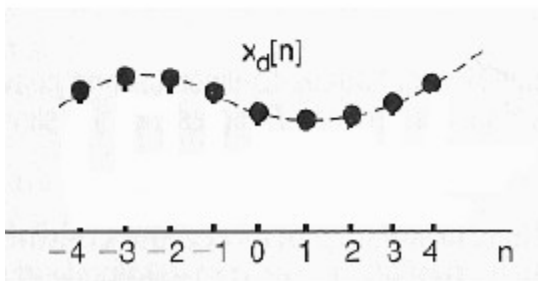
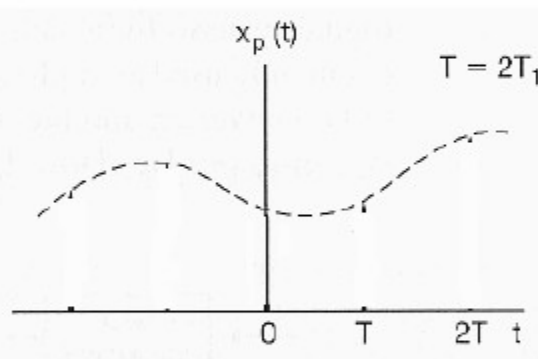
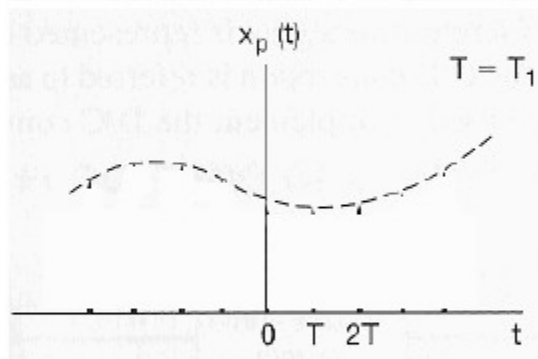
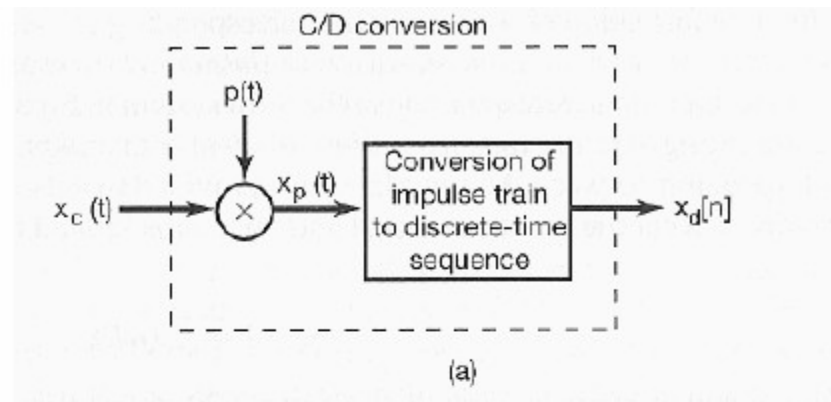
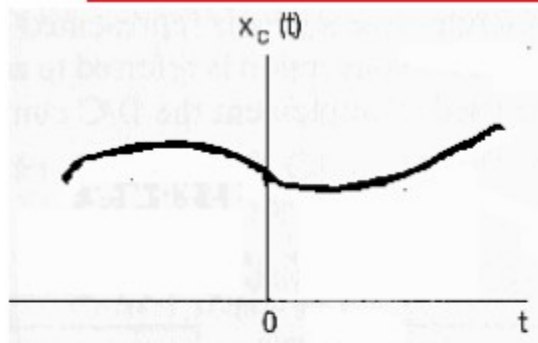
C/D: continuous-to-discrete-time conversion

A-to-D: analog-to-digital converter

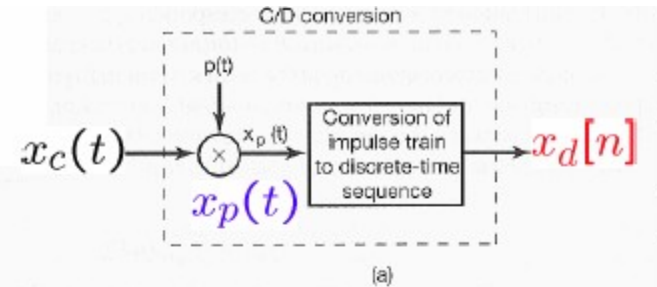
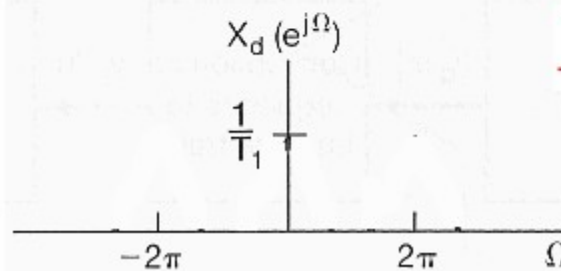
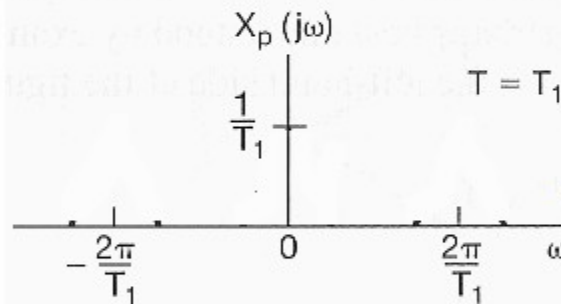
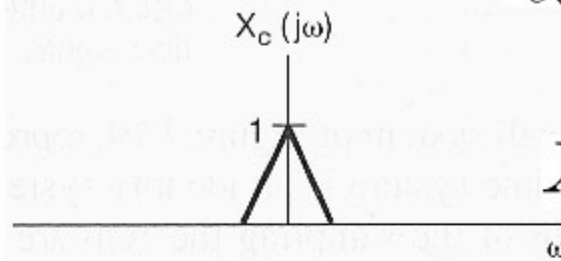
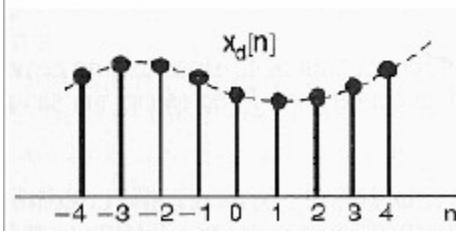
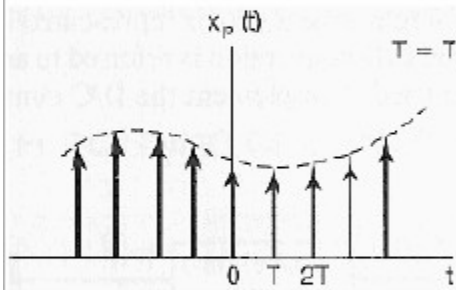
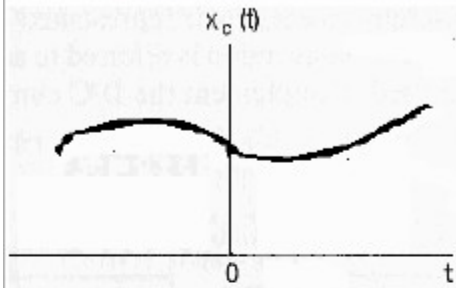
D/C: discrete-to-continuous-time conversion

D-to-A: digital-to-analog converter

■ C/D Conversion:



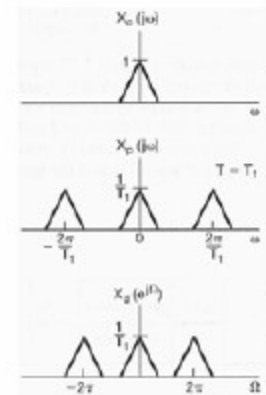
■ C/D Conversion:



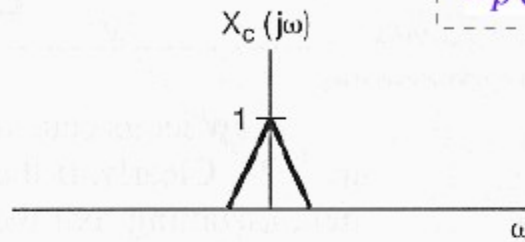
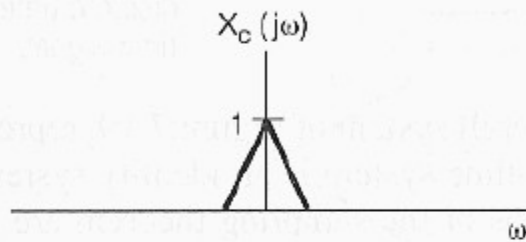
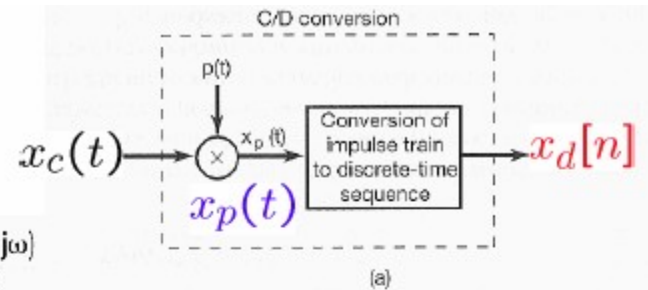
$$X_c(j\omega)$$

$$X_p(j\omega)$$

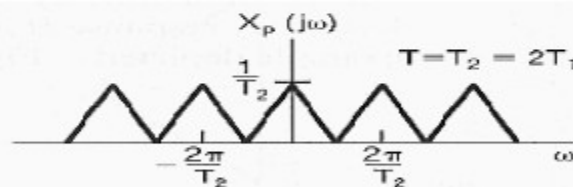
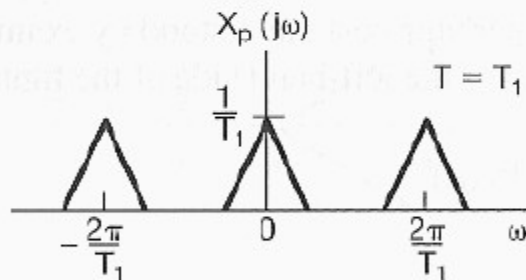
$$X_d(e^{j\Omega})$$



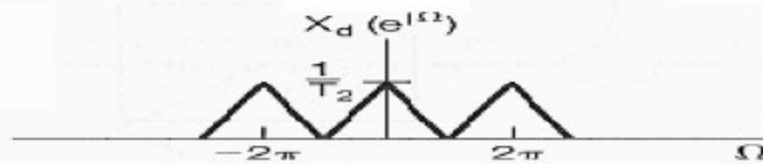
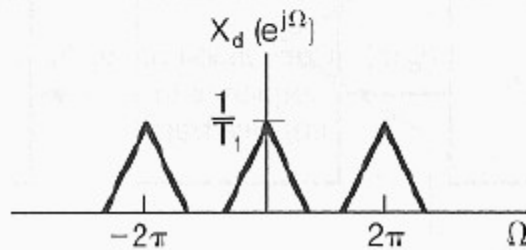
■ C/D Conversion:



$$X_c(j\omega)$$



$$X_p(j\omega)$$



$$X_d(e^{j\Omega})$$

■ C/D Conversion:

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x_c(nT)\delta(t - nT)$$

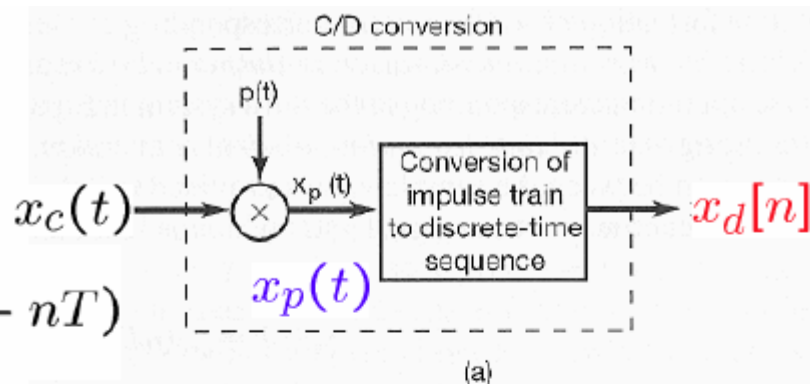


Table 4.2, p. 329

$$\delta(t - t_0) \xleftrightarrow{\mathcal{F}} e^{-j\omega t_0}$$

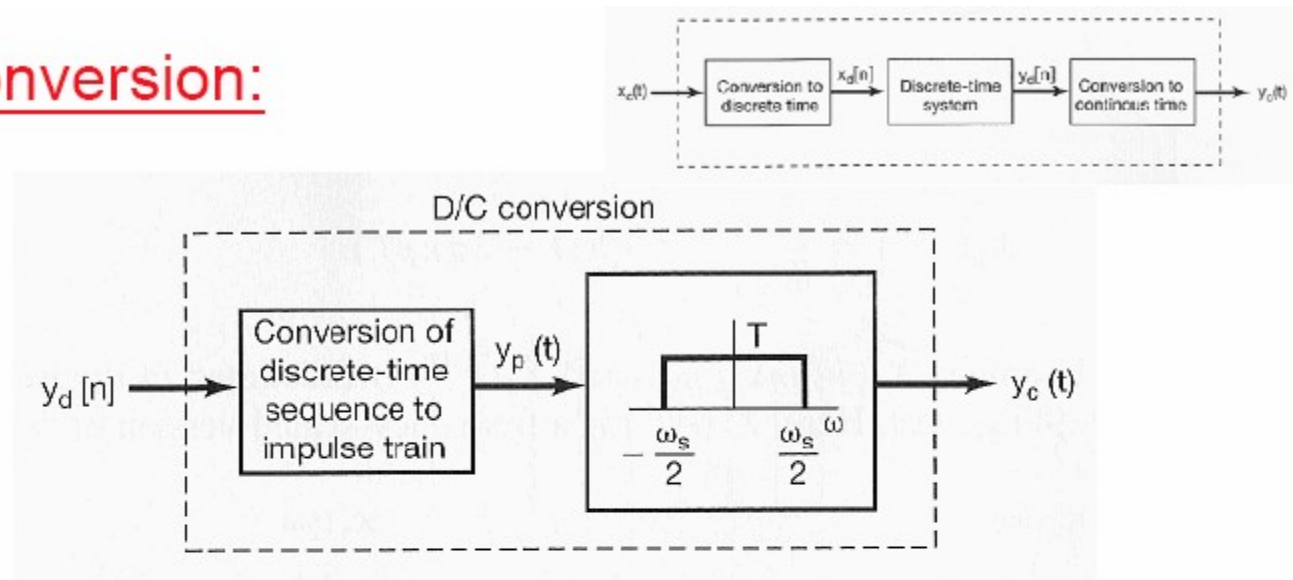
Eq 7.3, 7.6, p. 517

$$X_p(j\omega) = \sum_{n=-\infty}^{+\infty} x_c(nT)e^{-j\omega nT} = \frac{1}{T} \sum_{K=-\infty}^{+\infty} X_c(j(\omega - K\omega_s))$$

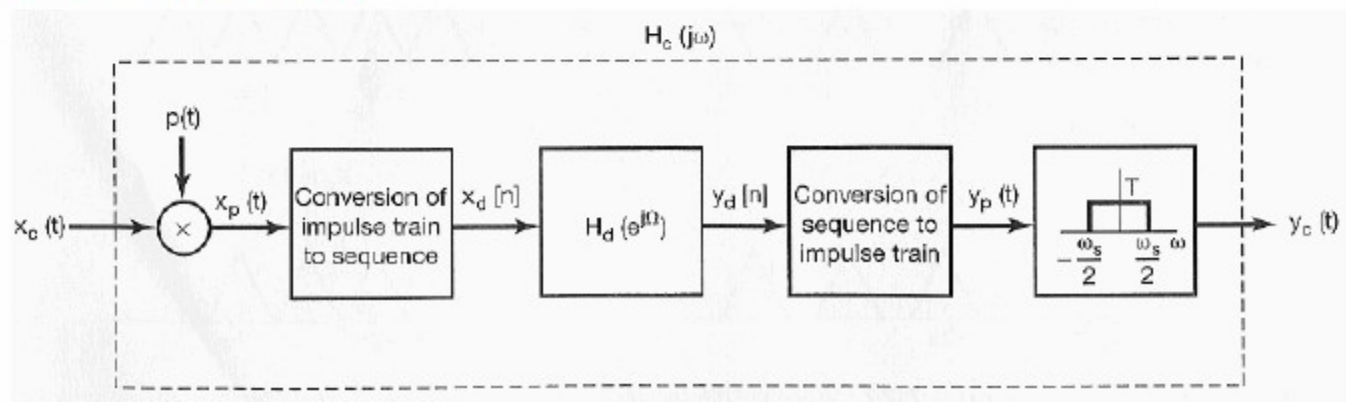
$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x_d[n]e^{-j\Omega n} = \sum_{n=-\infty}^{+\infty} x_c(nT)e^{-j\Omega n}$$

$$\Rightarrow X_d(e^{j\Omega}) = X_p\left(j\frac{\Omega}{T}\right) = \frac{1}{T} \sum_{K=-\infty}^{+\infty} X_c\left(j\left(\frac{\Omega}{T} - K\frac{2\pi}{T}\right)\right)$$

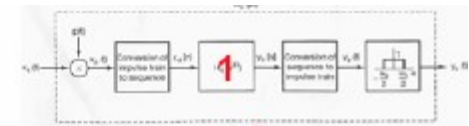
- D/C Conversion:



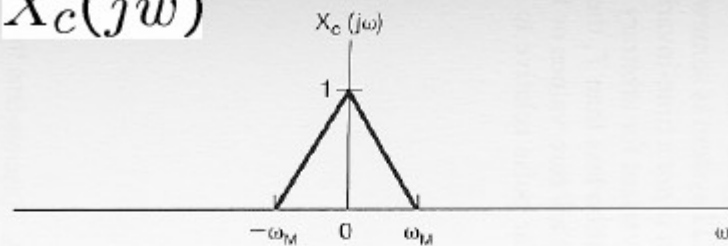
- Overall System:



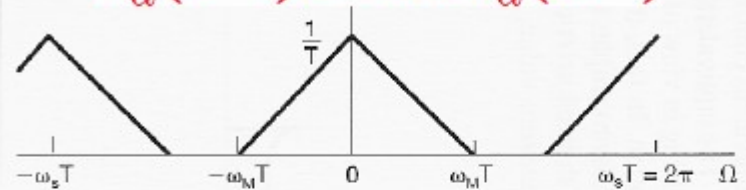
Frequency-Domain Illustration:



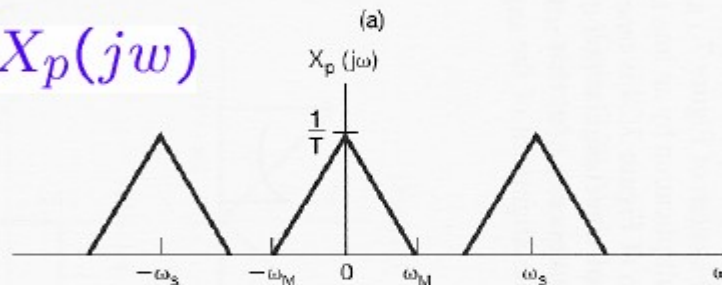
$$X_c(j\omega)$$



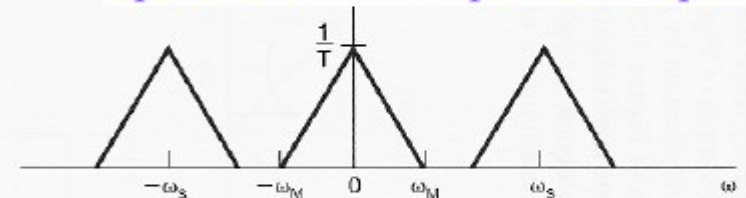
$$Y_d(e^{j\Omega}) = 1 X_d(e^{j\Omega})$$



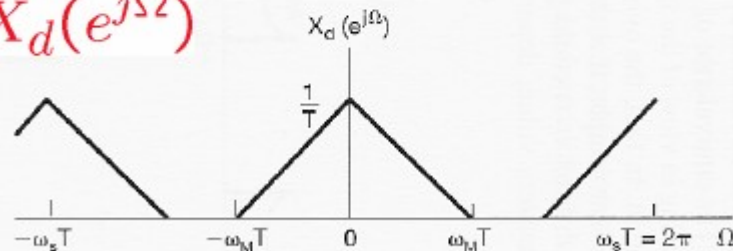
$$X_p(j\omega)$$



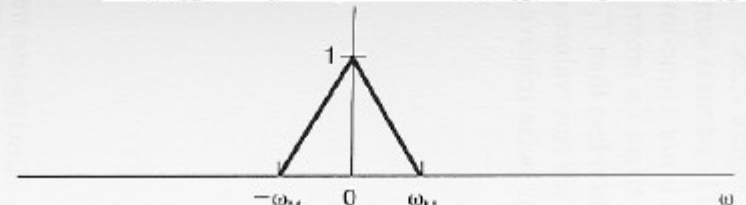
$$Y_p(j\omega) = H_p(j\omega) X_p(j\omega)$$



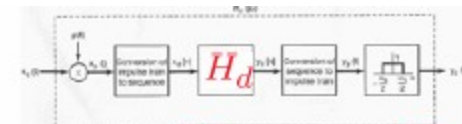
$$X_d(e^{j\Omega})$$



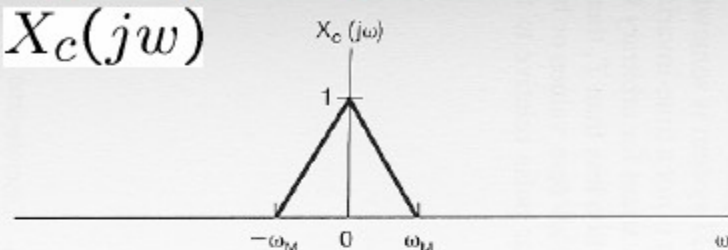
$$Y_c(j\omega) = H_c(j\omega) X_c(j\omega)$$



Frequency-Domain Illustration:

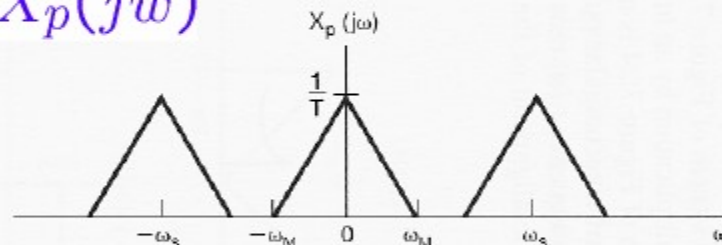


$$X_c(j\omega)$$



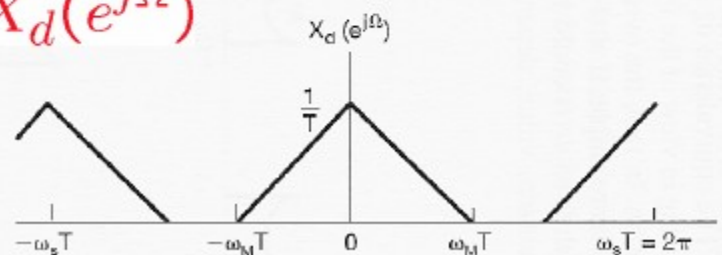
(a)

$$X_p(j\omega)$$



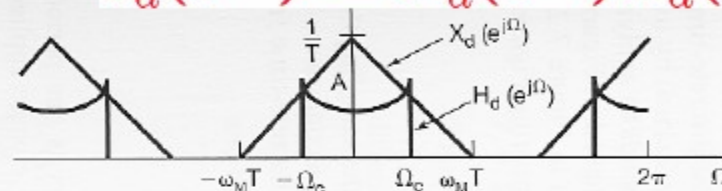
(b)

$$X_d(e^{j\Omega})$$



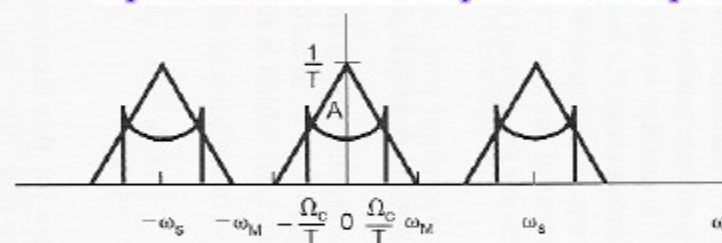
(c)

$$Y_d(e^{j\Omega}) = H_d(e^{j\Omega}) X_d(e^{j\Omega})$$



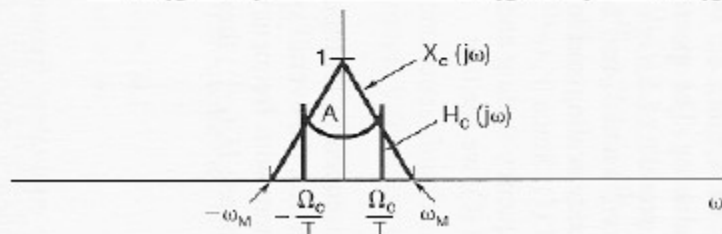
(d)

$$Y_p(j\omega) = H_p(j\omega) X_p(j\omega)$$



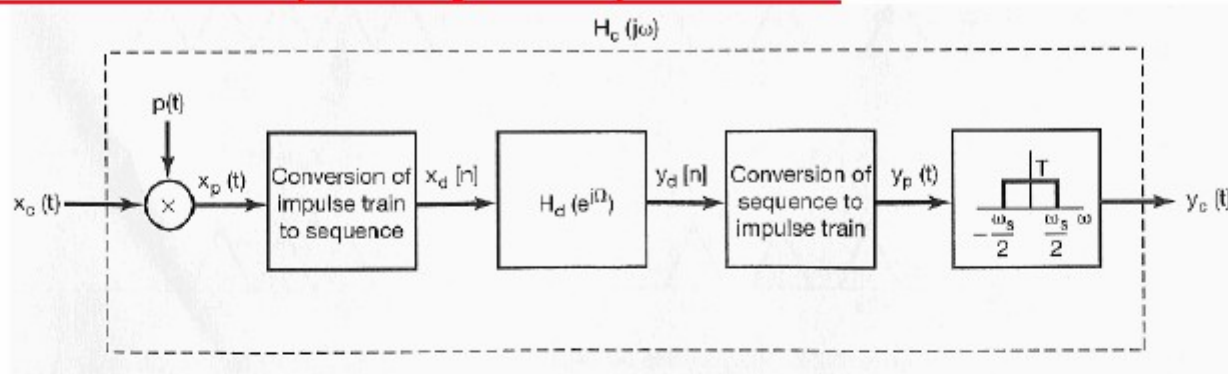
(e)

$$Y_c(j\omega) = H_c(j\omega) X_c(j\omega)$$



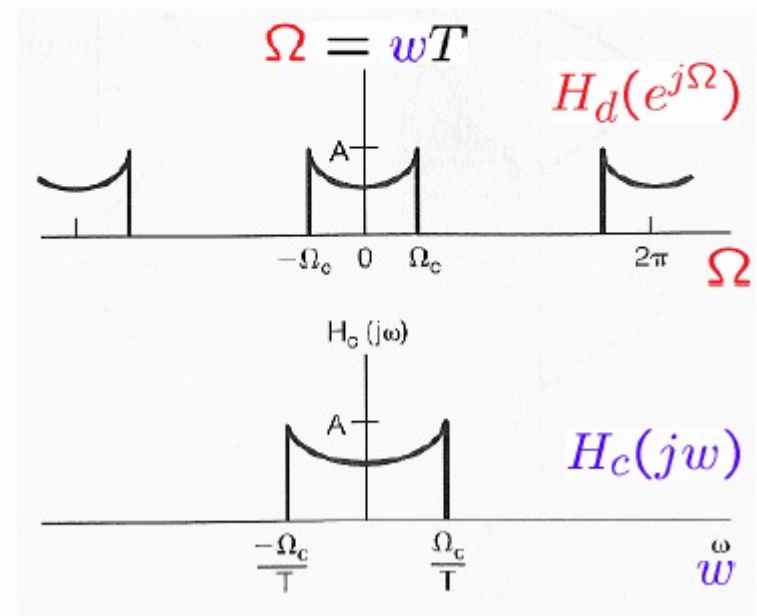
(f)

■ CT & DT Frequency Responses:



$$Y_c(j\omega) = X_c(j\omega)H_c(j\omega)$$

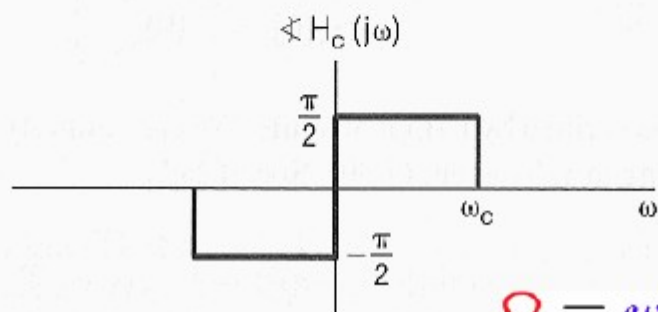
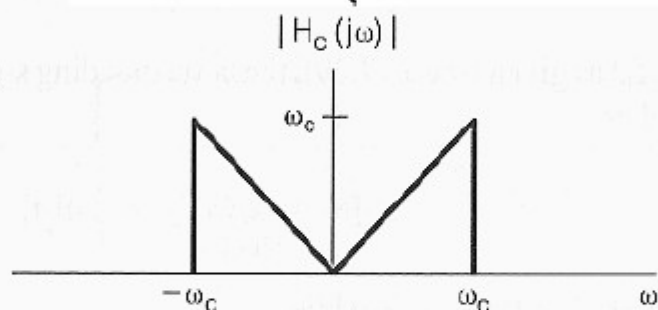
$$H_c(j\omega) = \begin{cases} H_d(e^{j\omega T}), & |\omega| < \omega_s/2 \\ 0, & |\omega| > \omega_s/2 \end{cases}$$



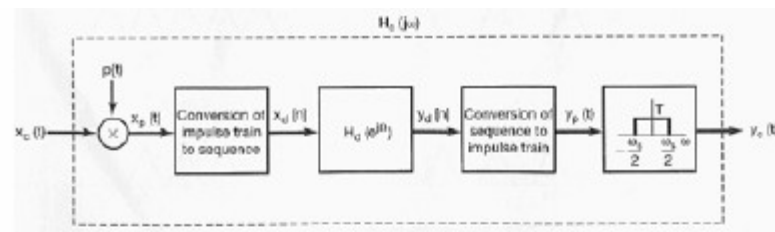
■ Digital Differentiator:

Ex 4.16, p. 317

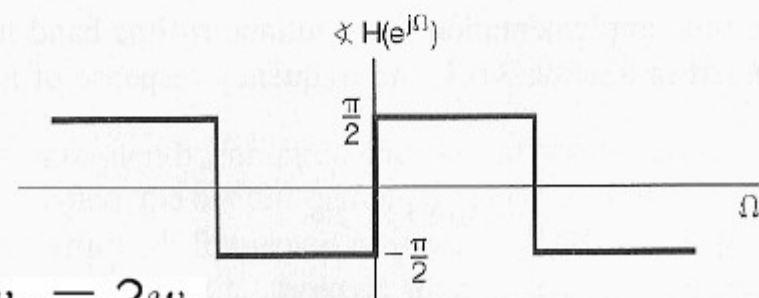
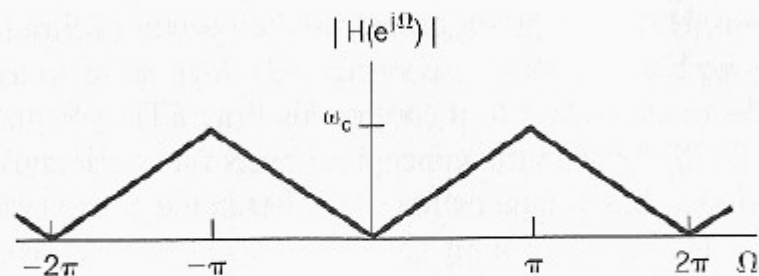
$$H_c(j\omega) = \begin{cases} j\omega, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$



$$\Omega = \omega T, \quad \omega_s = 2\omega_c$$



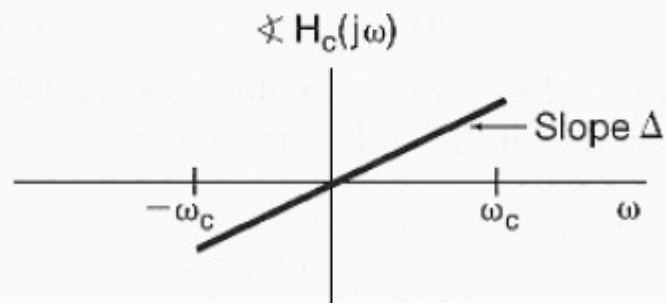
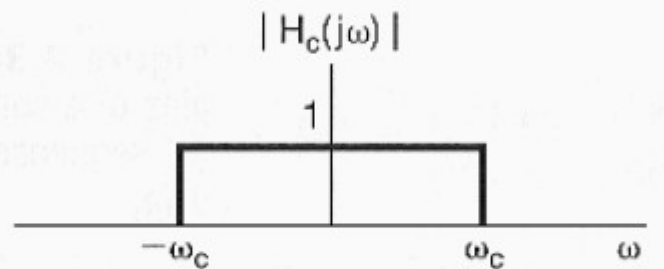
$$H_d(e^{j\Omega}) = j \left(\frac{\Omega}{T} \right), \quad |\Omega| < \pi$$



■ Half-Sample Delay:

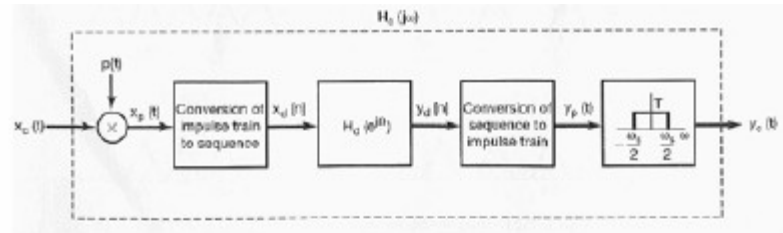
Ex 4.15, p. 317

$$H_c(j\omega) = \begin{cases} e^{-j\omega\Delta}, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$$

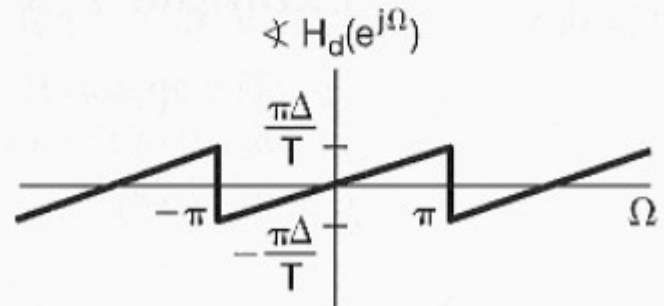
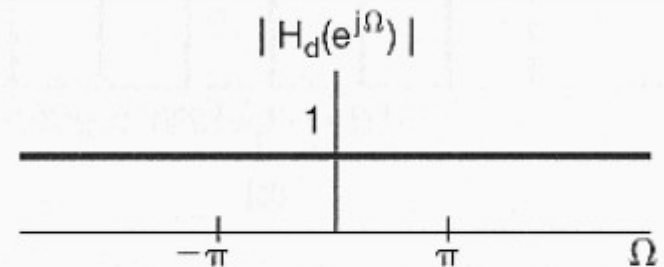


(a)

$$\Omega = \omega T, \quad \omega_s = 2\omega_c$$



$$H_d(e^{j\Omega}) = e^{-j\Omega\Delta/T}, \quad |\Omega| < \pi$$



(b)

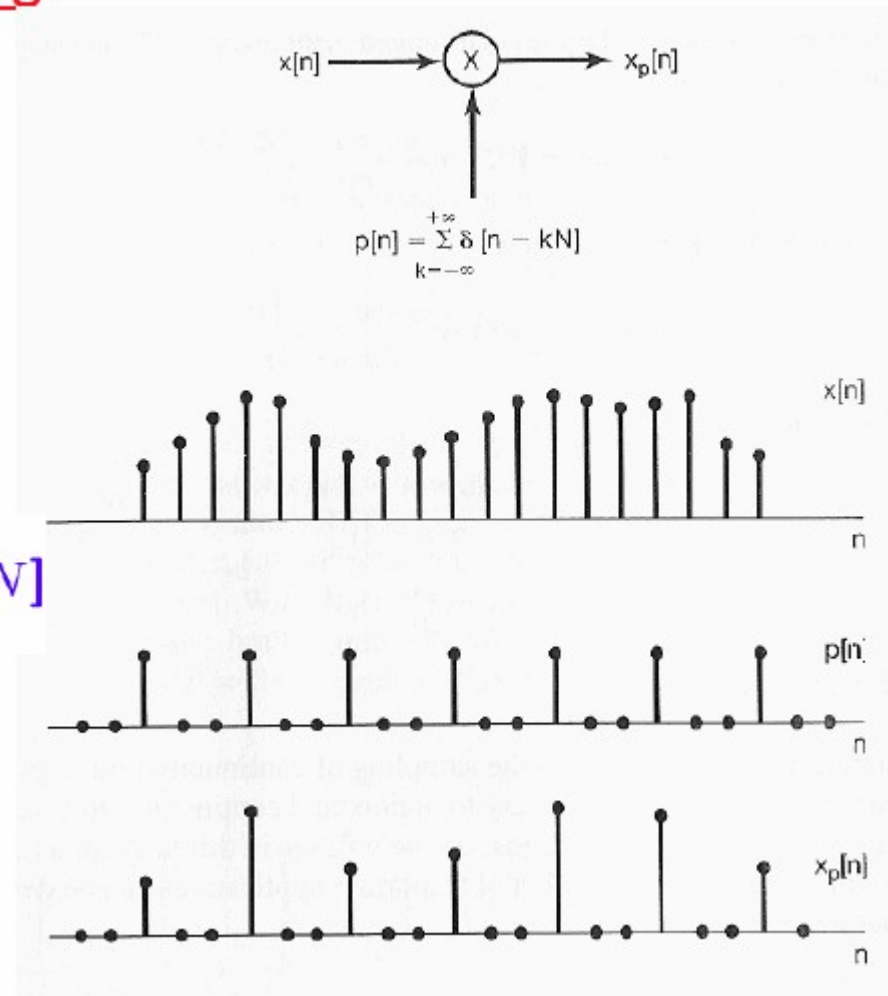
- Representation of of a Continuous-Time Signal by Its Samples: The Sampling Theorem
- Reconstruction of of a Signal from Its Samples Using Interpolation
- The Effect of Undersampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

■ Impulse-Train Sampling:

$$x_p[n] = \begin{cases} x[n], & \text{if } n = kN \\ 0, & \text{otherwise} \end{cases}$$

$$x_p[n] = x[n] p[n]$$

$$= \sum_{k=-\infty}^{+\infty} x[kN] \delta[n - kN]$$

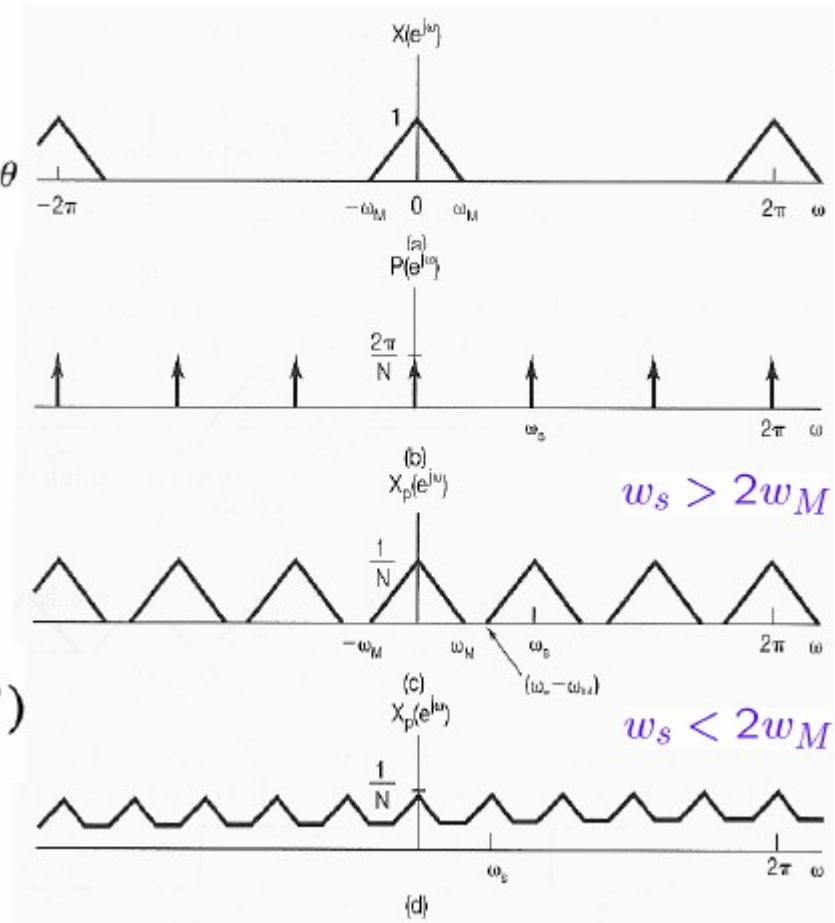


- Impulse-Train Sampling:

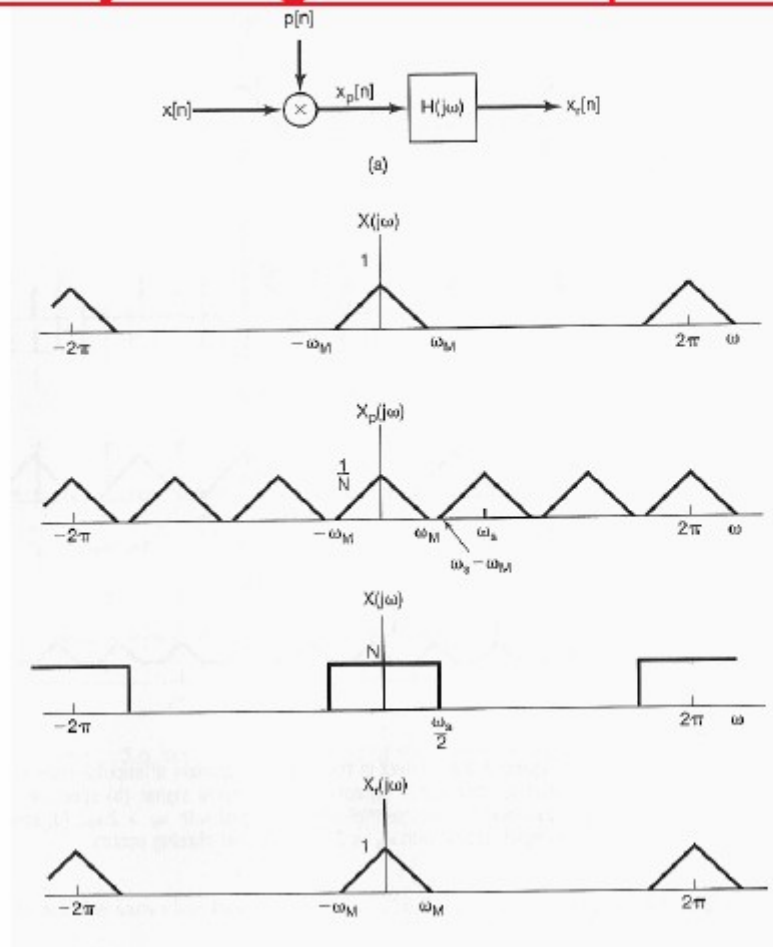
$$X_p(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} P(e^{j\theta}) X(e^{j(\omega-\theta)}) d\theta$$

$$P(e^{j\omega}) = \frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_s)$$

$$\Rightarrow X_p(e^{j\omega}) = \frac{1}{N} \sum_{k=0}^{N-1} X(e^{j(\omega - k\omega_s)})$$

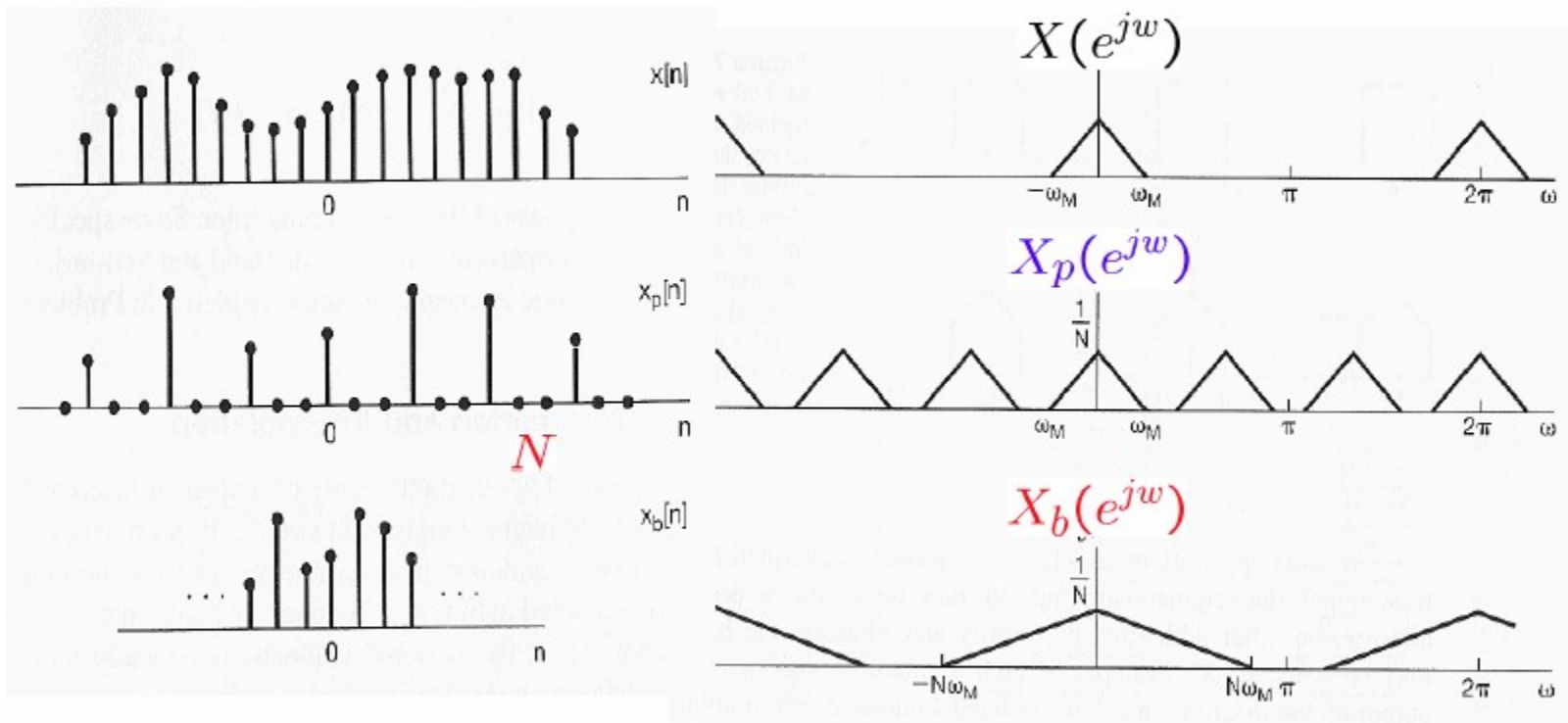


- Exact Recovery Using Ideal Lowpass Filter:



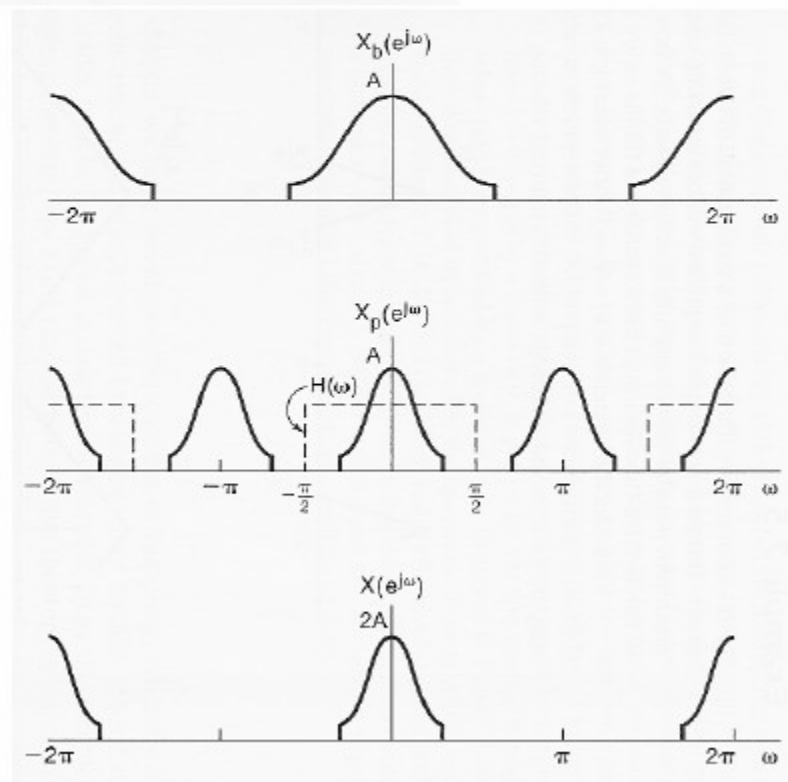
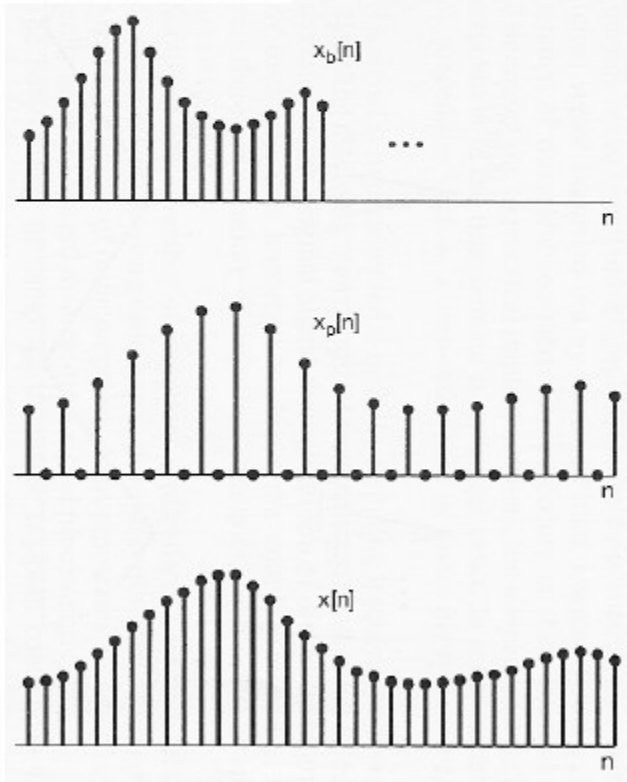
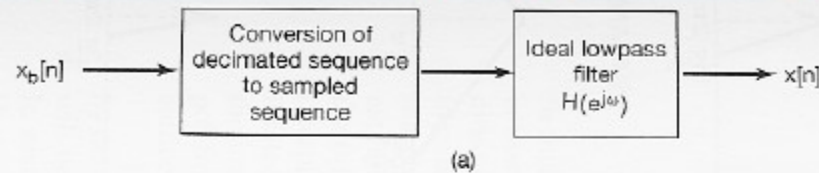
DT Decimation & Interpolation: Down-sampling

Eq 5.45, p. 378: Time expansion



$$X_b(e^{j\omega}) = X_p(e^{j\omega/N})$$

Higher Equivalent Sampling Rate: Up-sampling



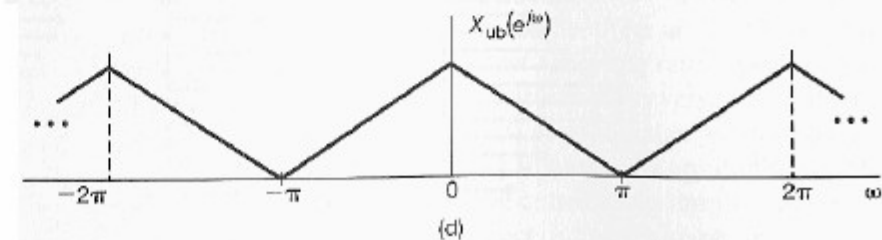
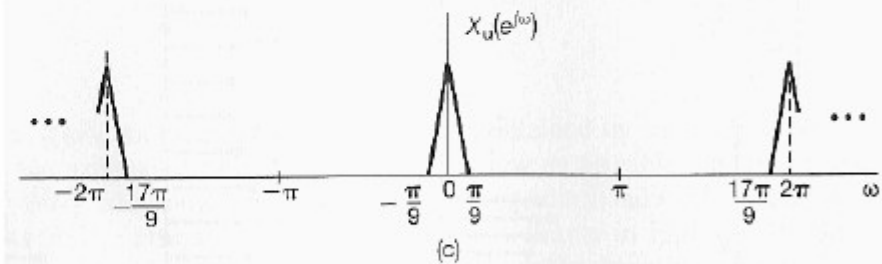
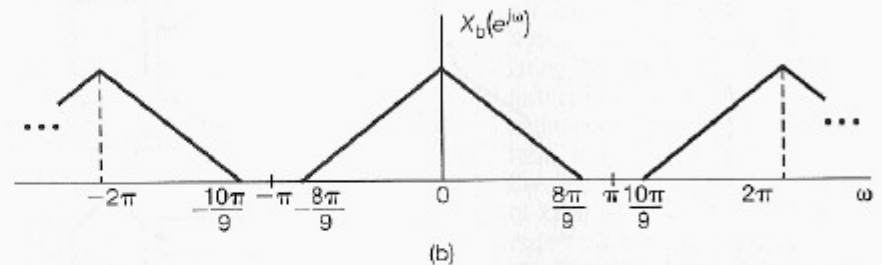
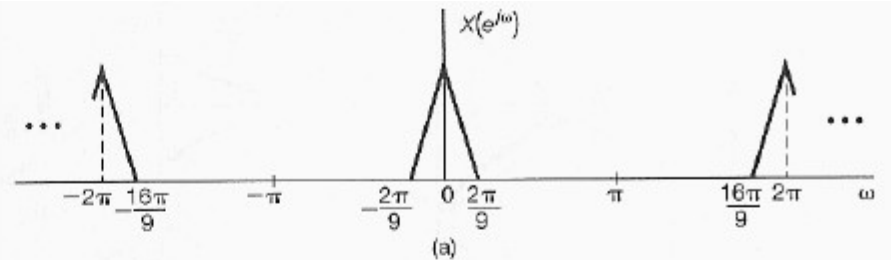
■ Down-sampling
+ Up-sampling:

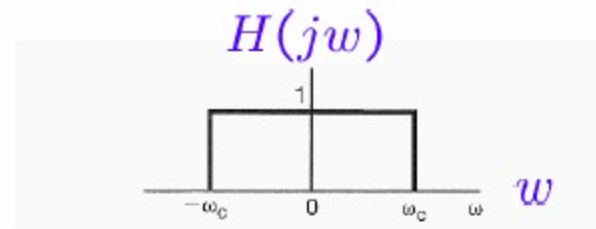
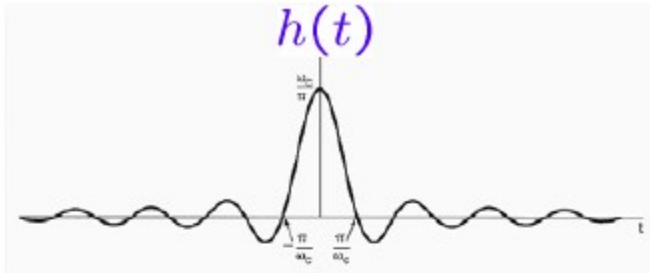
$$\frac{2\pi}{9} \times 4 < \pi$$

$$\frac{2\pi}{9} \times \frac{9}{2} = \pi$$

$$\frac{2\pi}{9} \times \frac{1}{2} = \frac{\pi}{9}$$

$$\frac{\pi}{9} \times 9 = \pi$$



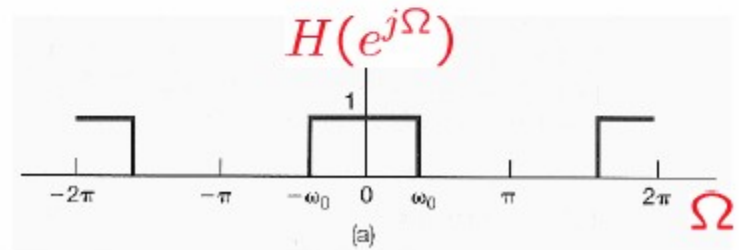
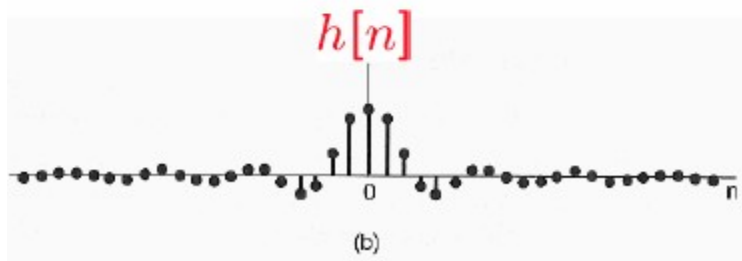


$$h(t) \xleftrightarrow{\text{C.T.F.T.}} H(jw)$$

$$\omega_s = \frac{2\pi}{T}$$

$$\Omega = wT$$

$$h[n] \xleftrightarrow{\text{D.T.F.T.}} H(e^{j\Omega})$$



■ Discrete-Time Processing of CT Signals

